

**MATH206A MULTIVARIABLE CALCULUS - PROF. P.
WONG**

EXAM I - SEPTEMBER 29, 2006

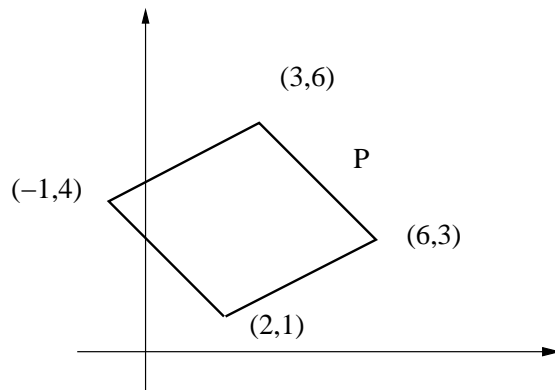
NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	Maximum Score	Your Score
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

1. Let P be the parallelogram in \mathbb{R}^2 with vertices $(2, 1)$, $(-1, 4)$, $(6, 3)$ and $(3, 6)$.



(5 pts) (i) Find the area of P .

(3 pts) (ii) Suppose $T(x_1, x_2) = (5x_1 + 4x_2, 5x_1 + 3x_2)$. Find the associated matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ where $\mathbf{x} = (x_1, x_2)$.

(4 pts) (iii) What are the vertices of the image $T(P)$?

(4 pts) (iv) What is the area of $T(P)$?

(4 pts) (v) What is the angle of the parallelogram P at the vertex $(6, 3)$? (You may express it in terms of inverse trig function.)

2. Let $\mathbf{a} = \mathbf{i} + \mathbf{j} - 6\mathbf{k}$, $\mathbf{b} = 4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, and $\mathbf{c} = -\mathbf{i} + 2\mathbf{k}$.

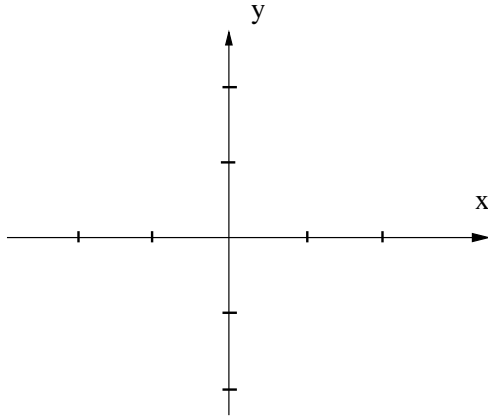
(7 pts) (i) Find a vector that is perpendicular to the plane spanned by \mathbf{a} and \mathbf{b} .

(7 pts) (ii) What is the volume of the parallelepiped formed by \mathbf{a} , \mathbf{b} , and \mathbf{c} ?

(6 pts) (iii) Find $\text{proj}_{\mathbf{c}}\mathbf{a}$.

3. Let $f(x, y) = x^2 + 2y^2 - 1$.

(5 pts) (i) Sketch the level curves of $f(x, y) = 3$, $f(x, y) = -1$, and $f(x, y) = -5$.



(5 pts) Describe or sketch the set of points in \mathbb{R}^3 that satisfy the equation $f(x, y) = x^2 + 2y^2 - 1$ (or the graph of $z = f(x, y)$)

(5 pts) (iii) What are the cylindrical coordinates of the point $(1, 2, 8)$?

(5 pts) (iv) Write the equation $z = x^2 + 2y^2 - 1$ in spherical coordinates.

4. The velocity of a particle in \mathbb{R}^3 is given by the parametrization

$$v(t) = \mathbf{i} + (1 + t)\mathbf{j} + \cos t\mathbf{k}.$$

(10 pts) (i) Find the position $r(t)$ of the particle with the initial point $r(0) = \mathbf{i} + \mathbf{k}$.

(10 pts) (ii) Give an equation (in vector form) of the line tangent to the path of the particle at the point $r(\pi)$.

5. (10 pts) (i) Consider the function

$$f(x, y) = \frac{xy}{x^2 + y^2}.$$

Determine whether the origin is a removable discontinuity of f . Justify your answer. [Hint: try approaching $(0, 0)$ from different directions]

(10 pts) (ii) Give an equation for the plane that is perpendicular to the line with parametric equations $x = 3t - 5$, $y = 7 - 2t$, $z = 8 - t$ and that contains the point $(1, -1, 2)$. [Hint: rewrite the line in vector form $\mathbf{x} = t\mathbf{m} + \mathbf{x}_0$]