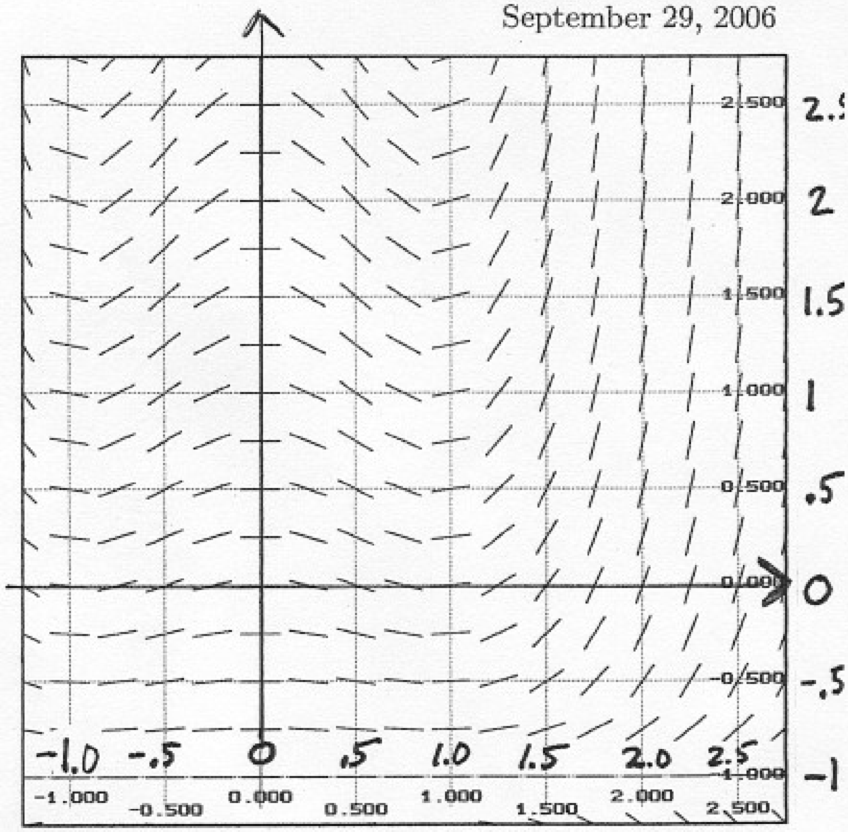
INITIALS______________

September 29, 2006



1. The slope field for the differential equation $dy/dt = (y+1)(t - \sin 2t)$ is drawn above. Using only a ruler and pencil (no calculator values), sketch the Euler method solution through the point $(-1, 1)$ as far as possible, using stepsize $h = 0.5$; your sketch will be a collection of big dots connected by line segments.

2. Consider the initial value problem $\begin{cases} dy/dt = (y+1)(t - \sin 2t) \\ y(0) = 2 \end{cases}$.

   (The solution is not the same as the one in problem 1 as it "starts" at a different point)

   2a. Find the find the value of C such that $y(t) = Ce^{0.5(t^2 + \cos 2t)} - 1$ is the solution of this IVP. Show all your work. (You don't have to verify $y(t)$ "satisfies" the DE; just find C)

   2b. Use your calculator to find the first four Euler approximations to the solution of the IVP in (2), using a step size of $h = 0.25$. Organize your results in a table, writing all results to *four* decimal places. Compare the exact value of $y(1)$ to its approximation in the table. How far apart are they?

3. Suppose the rate of change in population $c(t)$ in thousands of organisms per hour is measured and the results put into a table as follows:

time:	1:30	1:45	2:00	2:15	2:30	2:45	3:00	3:15	3:30	3:45	4:00
$c(t)$:	1.1	1.2	1.4	1.8	2.6	4.2	7.4	13.8	26.6	52.2	103.4

a. What does the integral $\int_{2:00}^{3:30} c(t)\,dt$ represent? (*Watch your limits!*)

b. Find each of the following estimates of the integral in (a) using only the information in the table. If the table doesn't supply the information needed, explain why not.

T_3

M_6

S_6

c. Is T_3 an under- or over-approximation of the integral in (a)? Explain!

d. If the population at 3:30 was 46 (thousand) what is your best estimate for the population at 2:00?

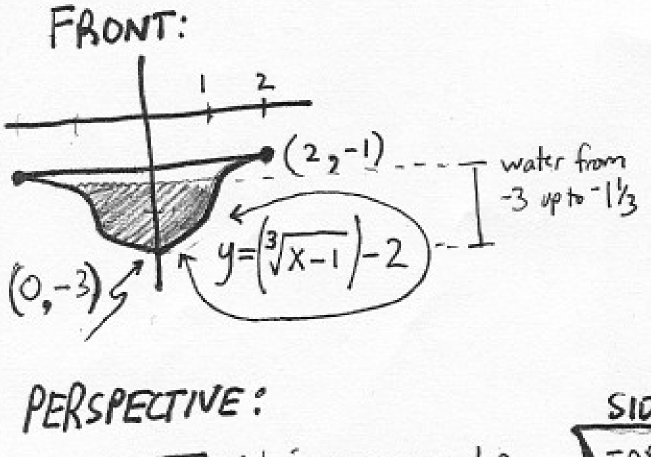
Use the method of substitution to find each of the following integrals. You may find the table useful, *if so*, tell which formula you used by its number.

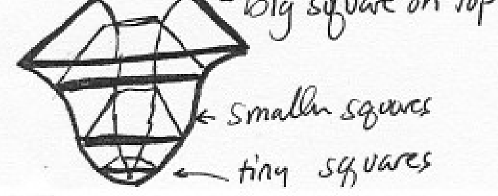
4. Use substitution to find the following integral. *Show the limits of integration* as they appear on the (new) integral once the substitution has been made. Leave your final answer in terms of e. $\int_{25}^{100} (e^{-\sqrt{x}}) / \sqrt{x}\, dx$

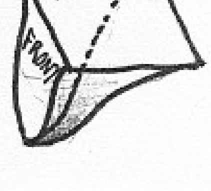
5. $\int \frac{\cos(5 \ln x)}{x} \ln x \, dx$

6. Let $I = \int_{0.25}^{1.0} e^{-\sqrt{x}}\,dx$. (This is not the same integral as in problem 4 and has nothing to do with it.) Use theorem 3 from section 6.2 to find the least n such that $|I - L_n| \leq 0.005$. In your work find (to three decimal places) the "best" possible K_1 as we've done in class. *Show all your work.* Then find L_n.

7. A water tank under ground has the shape shown here. The front face of the tank is a plane, horizontal cross sections behind the front are squares, so the tank is longer from front to back the higher you go. The top of the tank is 1 foot under the ground, and the water is 1 2/3 feet deep in the center at the front. What work is done to empty this tank, if the water is pumped up to 10' above ground level? Set up but don't evaluate the integral. Water weighs 62.4 lbs per ft^3.

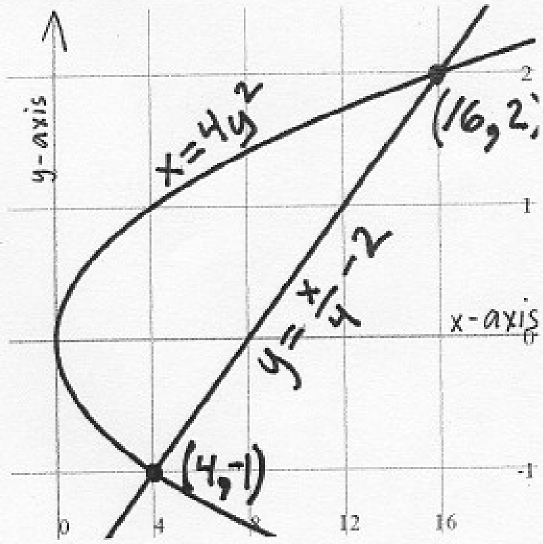






8. Let R be the region between the two curves shown here. In each part below, set up, but don't evaluate, an integral, or if necessary, a sum of two integrals, which represents the given quantity.

8a. The area of R, where the integral(s) is (are) in terms of dx.



8b. Again, the area of R, where the integral(s) is (are) now in terms of dy.

8c. The volume of the solid of revolution obtained by revolving the region R about the line ~~y~~ x = 20.