Math 205A   Quiz 03, page 1   September 28, 2007   NAME suggested solutions

1. Let S be the set of vectors $\{\mathbf{c}_1\, \mathbf{c}_2, \ldots, \mathbf{c}_k\}$, where each $\mathbf{c}_i$ is a column vector in $\mathbf{R}^m$. Some of the following statements are true (ie, they are theorems) and others are not. *Circle* each letter for which the statement is true, and mark an "X" through the letter if the statement is *false*.

a. If $m < k$ then S is linearly dependent.
~~b.~~ If $k < m$ then S is linearly independent.
c. If S contains the zero vector, S is linearly dependent.
~~d.~~ If S is linearly dependent, then every vector in S can be written as a linear combination of the other vectors in S.
~~e.~~ If at least one vector in S is not a linear combination of other vectors in S, then S is linearly independent.
f. The set S is linearly dependent $\iff$ at least one vector in S can be expressed as a linear combination of the others.
g. If the only solution to $A\mathbf{x} = \mathbf{0}$ is the trivial solution, where the members of S form the columns of A, then S is linearly independent.

2. By inspection, classify each of the following sets as linearly independent (LI) or linearly dependent (LD). Justify your answer by writing the letter corresponding to the theorem from problem 1 next to your answer of "LI" or "LD". For example, if you think the set in 2Q is linearly independent and statement **j** in problem 1 is your reason, then you'd write "LI-j" as your answer.

2A. $\left\{ \begin{bmatrix} 1 \\ -6 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ -7 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

LD - a

2B. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix} \right\}$

This matrix is in echelon form and shows every column is a pivot column. No free variables $\Rightarrow A\vec{x} = \vec{0}$ has only the trivial solution, thus its cols. are LI.

LI-g

2C. $\left\{ \begin{bmatrix} e^2+3 \\ \cos \pi/4 \end{bmatrix}, \begin{bmatrix} \sin 2\pi \\ e^0 - 1 \end{bmatrix} \right\}$

LD - c   because the second vector is $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

2D. $\left\{ \begin{bmatrix} 7 \\ 3 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ -5 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 12 \\ -2 \\ 7 \\ 9 \end{bmatrix} \right\}$

LD - f   it's clear that $\vec{c}_1 + \vec{c}_2 = \vec{c}_4$

3. Suppose the matrix A has column vectors $\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \mathbf{c}_4$ where A is: $\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 0 \\ 3 & 0 & 5 & 8 \end{bmatrix}$

3A: Express $\mathbf{c}_1$ as a linear combination of other column vectors or explain why you can't.

It's clear that $\vec{c}_1 + \vec{c}_3 = \vec{c}_4$, so $\vec{c}_1 = \vec{c}_4 - \vec{c}_3$.

3B: Express $\mathbf{c}_2$ as a linear combination of other column vectors or explain why you can't.

Any L.C. $x_1\vec{c}_1 + x_3\vec{c}_3 + x_4\vec{c}_4$ has the form $\begin{bmatrix} * \\ 0 \\ * \end{bmatrix}$, but $\vec{c}_2$ is NOT of this form, so it is not a L.C. of $\vec{c}_1$ $\vec{c}_3$ & $\vec{c}_4$.

3C: This example should explain why one of the statements in problem 1 above is false. *Which one?*

d

Since A consists of $K=4$ column vectors each of which is in $\mathbb{R}^m$ with $m=3$, we have $m<K$ so by 1(a), these columns are L.D. If (d) was true, every column would be a L.C. of the others, yet 3B shows this not to be the case. The example also shows (e) is false; either (d) or (e) is OK as the answer to this problem.