

Math 106 Fall 2007

Test 1 (60 points)

Name: _____

Show all your work to receive full credit for a problem.

Do not use the calculator integral function.

When you use a formula from the table of integrals, mention the formula number and the value(s) of any constant(s) that you may need.

Round off your answers to four decimal places.

Include units in your answers wherever possible.

There are eight questions. Questions are printed on both sides of a page.

You may use any of the following facts:

$$\text{Arclength} = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

$$|I - L_n| \leq \frac{K_1(b-a)^2}{2n}$$

$$|I - R_n| \leq \frac{K_1(b-a)^2}{2n}$$

$$|I - T_n| \leq \frac{K_2(b-a)^3}{12n^2}$$

$$|I - M_n| \leq \frac{K_2(b-a)^3}{24n^2}$$

Below are product rule, quotient rule and chain rule for derivatives.

$$(uv)' = u'v + uv'$$

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

$$(f \circ g)'(x) = f'(g(x))g'(x)$$

1. (8 points) Evaluate the following integral exactly. (You may use formulas 1-18 only from the table of integrals for this problem.)

$$\int \frac{6 \sin(2x)}{\cos^2(2x)} dx$$

2. (6 points) If $f(x)$ is increasing and concave up on the interval $[-1, 5]$, put the following quantities in ascending order. What feature of the graph of f helps you to put the quantities in ascending order?

$$M_{50}, T_{50}, \int_{-1}^5 f(x) dx$$

3. (10 points) Let $f(x) = e^x \ln x$ and let $I = \int_1^3 f(x) dx$.

(a) Use the error bound theorem to find a bound on the error committed by R_{20} . (Find the best possible value for K_1 .)

(b) Compute R_{20} using your calculator program. Use your answer in part (a) and the value of R_{20} to give an interval of possible values for I .

(c) What is the least value of n which guarantees that L_n approximates I within ± 0.1 ? Justify your answer.

4. (8 points) The rate of change of population of wolves in a park is given by $\frac{dP}{dt} = 0.05(P - 2000)$ where $P(t)$ is the population of wolves at time t .

(a) If there are 1500 wolves in the park at time $t = 0$, find an equation for the population of wolves at time t .

(b) What is the rate of change of population of the wolves when there are 1000 wolves in the park? Is the population increasing or decreasing at that instant? Explain.

5. (7 points) Find the exact length of the curve $y = 4 - 5x^2$ from $x = 0$ to $x = 1$.

6. (7 points) Sketch the region bounded by the curve $y = \frac{1}{x}$, and the lines $x = 1$, $x = 3$ and $y = 0$. Write (but do not evaluate) an integral to find the volume of the solid that is formed when the region is rotated about the line $y = 2$.

7. (7 points) A hemispherical tank of radius 7 ft is sitting on its flat surface which is at ground level. If the tank is filled with water (density = 62.4 lb/cubic foot) to a height of 4 ft, write (but do not evaluate) an integral equal to the work done in pumping all the water in the tank to a height of 12 ft above ground level.

8. (7 points) Use Euler's method with two steps to estimate the value of y at $x = 2$ if $y(1) = 3$ and $y' = x^2 - y^2$. (Do not use a calculator program for this problem.)