

NAME \_\_\_\_\_

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September 28  
2007

Mathematics 106a  
Calculus II  
Examination #1

Mr. Haines

(10) I. The graph of the function  $f(x) = x \ln x$  is increasing and concave up on the interval

$[1, 10]$ . Put the following quantities in increasing order:  $L_{100}$ ,  $R_{100}$ ,  $T_{100}$ ,  $M_{100}$ ,  $\int_1^{10} f(x) dx$ .

(30) II. Evaluate. [Your final answer should not contain any integrals]:

A.  $\int x^2 \cos(2x) dx$

B.  $\int \frac{\sin x \cos x}{36 + \sin^2 x} dx$

C.  $\int_0^1 \frac{x}{x^2 + 49} dx$

(10) III. Use Euler's method with four steps on the differential equation  $y' = y + 2t$  to estimate  $y(3.0)$  if  $y(1) = 0$ .

(10) IV. Write (but do not evaluate) an integral that gives the arc length of the graph of  $y = \ln(x)$  over the interval  $[1, 5]$ .

(10) V. Set up and evaluate an integral that gives the area between the graphs of  $y = \cos x$  and  $y = \sin x$  over the interval  $[0, \pi/4]$ . Do not approximate the area, but rather calculate it exactly.

(10) VI. Revolve about the  $x$ -axis the area under  $y = x^2$  and over  $[0, a]$ . The volume of this solid depends on the value of  $a$ . For what value of  $a$  is the volume equal to  $32\pi$ ?

- (10) VII. A rectangular tank full of cooking oil is sitting on the ground. A cubic foot of the oil weighs 57.4 lbs. The base of the tank is 5 feet by 10 feet and its height is 20 feet. Compute the amount of work required to pump the oil out of the tank to a point 30 feet above the top of the tank.

- (10) VIII. Find the solution of the initial value problem:

$$y' = x^2 y \text{ with } y(0) = 10 .$$