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September 28
2007

Mathematics 106a
Calculus II
Examination #1

Mr. Haines

(10) I. The graph of the function $f(x) = x \ln x$ is increasing and concave up on the interval

$[1, 10]$. Put the following quantities in increasing order: L_{100} , R_{100} , T_{100} , M_{100} , $\int_1^{10} f(x) dx$.

(30) II. Evaluate. [Your final answer should not contain any integrals]:

A. $\int x^2 \cos(2x) dx$

B. $\int \frac{\sin x \cos x}{36 + \sin^2 x} dx$

C. $\int_0^1 \frac{x}{x^2 + 49} dx$

(10) III. Use Euler's method with four steps on the differential equation $y' = y + 2t$ to estimate $y(3.0)$ if $y(1) = 0$.

(10) IV. Write (but do not evaluate) an integral that gives the arc length of the graph of $y = \ln(x)$ over the interval $[1, 5]$.

(10) V. Set up and evaluate an integral that gives the area between the graphs of $y = \cos x$ and $y = \sin x$ over the interval $[0, \pi/4]$. Do not approximate the area, but rather calculate it exactly.

(10) VI. Revolve about the x -axis the area under $y = x^2$ and over $[0, a]$. The volume of this solid depends on the value of a . For what value of a is the volume equal to 32π ?

- (10) VII. A rectangular tank full of cooking oil is sitting on the ground. A cubic foot of the oil weighs 57.4 lbs. The base of the tank is 5 feet by 10 feet and its height is 20 feet. Compute the amount of work required to pump the oil out of the tank to a point 30 feet above the top of the tank.

- (10) VIII. Find the solution of the initial value problem:

$$y' = x^2 y \text{ with } y(0) = 10 .$$