NAME____

I___II___III___IV___V___VI___VII___VIII___ IX___X___XI___TOTAL ___________

September 27 2004

Mathematics 205
Linear Algebra
Examination #1

Mr. Haines

(10) I. Suppose $\mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and the columns of the matrix A are $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ 4 \\ 4 \end{bmatrix}$.

A. Determine whether or not the equation $A\mathbf{x} = \mathbf{b}$ has a solution and explain your reasoning.

B. Determine whether or not $\mathbf{b}$ is in the span of the columns of A and explain your reasoning.

(10) II. Give an example of a linear system of equations in two variables whose solution set is a straight line.

(5) III. Give a parametric vector equation of the line through the

point $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ parallel to the vector $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$.

(5) IV. If the matrix A = $\begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -6 \end{bmatrix}$, $T(\mathbf{x}) = A\mathbf{x}$, and $\mathbf{e_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, what is

$T(\mathbf{e_3})$?

(20) V. T is a function with rule $T(x_1, x_2, x_3, x_4, x_5) = (x_1 + x_3, 0, x_1 + x_5, x_2 - x_4)$.

A. What is the domain of T ?

B. What is the codomain of T ?

C. Is T one-to-one? Explain your answer.

D. Is T onto? Explain your answer.

(10) VI. Suppose an economy has two sectors, Goods and Services. Each year, Goods sells 40% of its output to services and keeps the rest, while Services sells 70% of its output to Goods and keeps the rest. Find equilibrium prices for the annual outputs of the Goods and Services sectors that make each sector's income match its expenditures.

(10) VII. If $T : \mathfrak{R}^2 \rightarrow \mathfrak{R}^2$ reflects points through the line $x_1 = x_2$, give the standard matrix of the linear transformation T.

(10) VIII. The <u>augmented</u> matrix of a linear system has been reduced by row operations to the form

$$\begin{bmatrix} 1 & -1 & 0 & 3 & -2 \\ 0 & 1 & 0 & -4 & 7 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduce this augmented matrix reduced row echelon form and describe the solution set of the original system as a parametric vector equation.

(5) IX. Suppose $T(\mathbf{x}) = A\mathbf{x}$, where $A = \begin{bmatrix} 0 & 3 & -6 & 6 & 0 & -5 \\ 3 & -7 & 8 & -5 & 3 & 9 \\ 3 & -9 & 12 & -9 & 3 & 15 \end{bmatrix}$

Without reducing this matrix to reduced echelon form, explain why T is not one-to-one.

(10) X. Suppose $A = \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Also suppose that $T(\mathbf{x}) = A\mathbf{x}$.

Find two vectors $\mathbf{x_1}$, and $\mathbf{x_2}$ whose image under T is $\mathbf{b}$.

(5) XI. TRUE OR FALSE? (Don't guess! The number of incorrect responses will be subtracted from the number of correct ones. Thus, random guessing earns you no points at all.)

________ 1. If A is a 2×3 matrix, then the transformation T with rule $T(\mathbf{x}) = A\mathbf{x}$ cannot be one-to-one.

________ 2. Every linear transformation from $\Re^m$ to $\Re^n$ can be expressed as a matrix transformation.

________ 3. The set Span $\{\mathbf{u}, \mathbf{v}\}$ is always visualized as a plane through the origin.

________ 4. A linear transformation is a special kind of function.

_______ 5. If $\mathbf{v_1}$ is a multiple of $\mathbf{v_2}$, then the set $\{\mathbf{v_1}, \mathbf{v_2}\}$ is linearly independent.