1. Suppose $x_3 \begin{bmatrix} -8 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 4 \\ 0 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}$, where x_3 and x_5 are free, is the solution $\mathbf{v}_h$ of the homogeneous matrix equation $B\mathbf{x} = \mathbf{0}$ for some matrix B. Also, let $\mathbf{v}_1$, $\mathbf{v}_2$, $\ldots$, be the column vectors of B.

1A. You can *not* tell from the above info how many rows B has. But how many *columns* must B have, and how do you know?

1B. Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \ldots\}$ of column vectors of B linearly independent? Explain in terms of the definition of LI (that is, consider the solutions of $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots = \mathbf{0}$).

1C. Use the equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots = \mathbf{0}$ to express $\mathbf{v}_1$ as a specific linear combination of the other column vectors, or explain why this is impossible.

1D. Express $\mathbf{v}_2$ as a linear combination of the other column vectors, or explain why this is impossible.

1E. Let $\mathbf{b} = 7\mathbf{v}_1 + 6\mathbf{v}_2 - 12\mathbf{v}_4$ in the following two questions:

1E (i). Express $\mathbf{b}$ as a linear combination of the column vectors of B *without* using $\mathbf{v}_4$ (by replacing $\mathbf{v}_4$ with a LC of the other column vectors).

1E (ii). Can you express $\mathbf{b}$ as a linear combination of the column vectors of B *without* using $\mathbf{v}_2$? Explain your answer.

1 Challenge Bonus Question: Let m be the number of rows of B. Suppose $B\mathbf{x} = \mathbf{c}$ does *not* have a solution for every $\mathbf{c}$ in $\mathbf{R}^m$. What is the RREF of B, where m is as small as possible?