

### Math 106: Review for Exam I

1. Find the following. [Substitution tip: usually let  $u =$  a function that's "inside" another function, especially if  $du$  (possibly off by a multiplying constant) is also present in the integrand.]

(a)  $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

(b)  $\int_{\pi}^{2\pi} \cos^7(5x) \sin(5x) dx$

(c)  $\int \frac{7x^2}{1+x^6} dx$

(d)  $\int \cos^3(5x) dx$

(e)  $\int_6^{10} x\sqrt{10-x} dx$

2. If  $f(x)$  is decreasing and concave up, put the following quantities in ascending order.

$$L_{100}, R_{100}, T_{100}, M_{100}, \int_a^b f(x) dx$$

What can you say with certainty about where  $S_{200}$  would fit into your list above? [8:00 and 9:30 sections may omit this part.]

3. Suppose  $f(t)$  is the rate of change (in animals per month) of a population  $P(t)$ .

(a) What does  $\int_4^{12} f(t) dt$  represent in this problem?

(b) Find the best possible left, right, midpoint, trapezoidal, and Simpson's approximations to  $\int_4^{12} f(t) dt$  given the data in the table below. [8:00 and 9:30 sections may omit Simpson's approximation.]

$t$	4	6	8	10	12
$f(t)$	15	11	8	4	3

4. Find bounds for each of the following errors if  $I = \int_2^7 \ln x dx$ .

(a)  $|I - L_{100}|$

(b)  $|I - T_{100}|$

(c)  $|I - M_{100}|$

5. Use Euler's method with three steps on the differential equation  $\frac{dy}{dt} = y - t$  to estimate  $y(2.5)$  if  $y(1) = 0$ .

6. Solve the differential equation  $dy/dx = 2xy + 6x$  if the solution passes through  $(0, 5)$ .

7. Write integrals equal to

(a) the arc length of  $y = x^2$  on the interval  $[1, 5]$

(b) the area bounded by  $y = x^2 - 8x + 24$  and  $y = 3x$

8. Consider the region bounded by  $y = \sqrt{x}$ ,  $y = 0$ , and  $x = 9$ . Write an integral equal to the volume generated if this region is rotated about

(a) the  $x$ -axis

(b) the line  $x = -1$

9. A pyramid has a square base 30 feet to a side and a height of 10 feet. Write integrals equal to

(a) the volume of the pyramid [8:00 and 9:30 sections may omit this part, though much of it will be repeated in part(b).]

(b) the work done in pumping all the fluid to a point 5 feet above the pyramid if the pyramid is filled to a height of 8 feet with water (62.4 pounds per cubic foot)