

1. Let \mathbf{b} and $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ be vectors in \mathbf{R}^m . Complete the following sentence so that it gives the definition of *linear combination*: “We say \mathbf{b} is a linear combination of the the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ if and only if...”

2. Let $\mathbf{a}_1 = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 6 \\ 3 \\ -9 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$, $\mathbf{a}_4 = \begin{bmatrix} 10 \\ 4 \\ -4 \end{bmatrix}$. Also, let $\mathbf{b} = \begin{bmatrix} 16 \\ 8 \\ 26 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} 26 \\ 8 \\ 16 \end{bmatrix}$.

2A. Is \mathbf{b} in the span of $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$? Explain your answer. Show any matrices and corresponding rref's you use.

2B. Let A be the matrix whose columns are $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ and \mathbf{a}_4 . Express all solutions of $A\mathbf{x} = \mathbf{c}$ in parametric vector form, that is, as $\mathbf{p} + \mathbf{v}_h$ where \mathbf{p} is a particular solution of $A\mathbf{x} = \mathbf{c}$ and \mathbf{v}_h is all solutions of the corresponding homogeneous equation $A\mathbf{x} = \mathbf{0}$. Show any relevant matrices used in your work.

2C. Use your work in (2B) to find two nontrivial solutions \mathbf{s}_1 and \mathbf{s}_2 of $A\mathbf{x} = \mathbf{0}$. CIRCLE your answers.

2D. Now let T be the matrix whose columns are $\mathbf{a}_1, \mathbf{a}_3$ and \mathbf{a}_4 (so T looks like A if you take out A 's second column). What is v_h now? That is, what are the solutions of $T\mathbf{x} = \mathbf{0}$?