

NAME _____

I _____ II _____ III _____ IV _____ V _____ VI _____ VII _____ TOTAL _____

September 25
2003

Mathematics 206a
Multivariable Calculus
Examination #1

Mr. Haines

(10) I. Give a coordinate equation for the plane through the point $(3, 4, 5)$ which is perpendicular to the vector $\mathbf{n} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$.

(10) II. Suppose \mathbf{a} is a vector with tail at the point $(3, 1, -2)$ and head at the point $(5, 4, 3)$.
Give a unit vector that is perpendicular to \mathbf{a} .

(36) III. If $\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ then

A. $\text{comp}_{\mathbf{b}}(\mathbf{a}) =$

B. $\text{proj}_{\mathbf{b}}(\mathbf{a}) =$

C. $\mathbf{a} \times \mathbf{b} =$

D. $\mathbf{b} \times \mathbf{b} =$

E. $\mathbf{b} \times \mathbf{a} =$

F. A parametrization for the line through the point $(1, 1, 1)$ and parallel to \mathbf{a} is:

G. A parametrization for the plane through the point $(1, 1, 1)$ and parallel to \mathbf{a} and \mathbf{b} is:

H. The cosine of the angle between \mathbf{a} and \mathbf{b} is:

I. $\|\mathbf{a}\| =$

(10) IV. Calculate the integral:

$$\int \left(\frac{1}{t} \mathbf{i} + (\cos t) \mathbf{j} + \sqrt{t} \mathbf{k} \right) dt$$

(16) V. Give examples of:

A. Two unit vectors in \mathfrak{R}^3 that are perpendicular.

B. Equations of two distinct parallel planes in \mathfrak{R}^3 .

C. Parametric equations of two distinct parallel lines in \mathfrak{R}^3 .

D. A path in \mathfrak{R}^3 , i.e. a function $\mathbf{f} : \mathfrak{R} \rightarrow \mathfrak{R}^3$, which crosses itself, i.e. $\mathbf{f}(a) = \mathbf{f}(b)$ for some numbers a and b .

(10) VI. Suppose $\mathbf{A} = \begin{bmatrix} 6 & 4 \\ 6 & 3 \end{bmatrix}$ and $\mathbf{T} : \mathfrak{R}^2 \rightarrow \mathfrak{R}^2$ is a linear transformation with the formula: $\mathbf{T}(\mathbf{x}) = \mathbf{Ax}$.

If P is the parallelogram in \mathfrak{R}^2 with vertices $(1, 2)$, $(4, -1)$, $(3, 6)$, and $(6, 3)$,

A) What is the area of P ?

B) What is the area of $\mathbf{T}(P)$?

(8)VII. Given the quadratic form $r(x, y, z) = x^2 - 2xy + 3y^2 + 2z^2$

A. Express $r(x, y, z)$ in the form $(x, y, z)\mathbf{S}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, where \mathbf{S} is a symmetric matrix

B. Is $r(x, y, z)$ positive definite, negative definite, or indefinite? Explain why.