

MATH 205A • Fall 2005

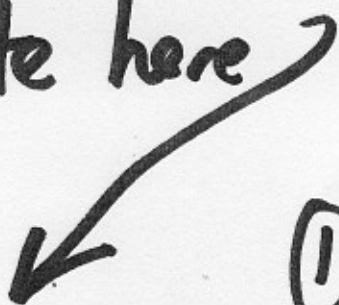
NAME

MINI-EXAM

9/23/2005

ROSS -  
suggested solns.

Do NOT write here



1
2
3
4
TOTAL

① show ALL your work!

② READ the questions!

③ BE NEAT!

GOOD Luck!

1. Suppose the augmented matrix corresponding to some matrix equation  $Ax = b$  reduces to the following:

$$\left[ \begin{array}{cccc|ccc} 1 & 2 & 0 & 0 & 5 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 & 4 & 0 & 0 & t \\ 0 & 0 & 0 & 1 & -3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r \end{array} \right].$$

1a. What (if any) conditions are there on  $r$ ,  $s$ , and  $t$  so that ...

i. there are no solutions to  $Ax = b$ .

There will be no solutions iff  $r \neq 0$ ; the values of  $s$  &  $t$  do not matter.

ii. there is exactly one solution to  $Ax = b$ .

There can never be exactly one. The existence of free variables means there are either no solns (if the system is inconsistent), or

iii. there are infinitely many solutions to  $Ax = b$ .

infinitely many solns.  
There are infinitely many iff  $r = 0$ ; the values of  $s$  and  $t$  can be any real numbers (no restrictions)

1b. Write the solution to this system (given that there are solutions) in the form  $\vec{v}_h + \vec{p}$  as done in class, that is, where  $\vec{v}_h$  represents all solutions of the homogeneous system  $Ax = 0$  and  $\vec{p}$  is a particular solution to  $Ax = b$ .

$$\text{We have } \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -2x_2 - 5x_5 + 10 \\ x_2 \\ -4x_5 + t \\ 3x_5 + 8 \\ x_5 \\ x_6 \\ s \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -5 \\ 0 \\ -4 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ t \\ 8 \\ 0 \\ 0 \\ s \end{bmatrix}$$

where  $x_2$ ,  $x_5$ , and  $x_6$  are free.

(note  $\vec{v}_h + \vec{p}$ )

2. Consider the system of equations

$$x_1 + 2x_2 + 7x_3 = 4$$

$$2x_1 + 5x_2 + 19x_3 = 6$$

$$4x_1 + 5x_2 + 13x_3 = h$$

What (if any) conditions on  $h$  guarantee a solution exists? Show all your work.

The augmented matrix corresponding to this system is  $\left[ \begin{array}{ccc|c} 1 & 2 & 7 & 4 \\ 2 & 5 & 19 & 6 \\ 4 & 5 & 13 & h \end{array} \right]$

Careful row reduction shows it equivalent to  $\left[ \begin{array}{ccc|c} 1 & 2 & 7 & 4 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 0 & h-22 \end{array} \right]$ ,

which has a sol'n (infinitely many, in fact) iff  $h-22=0$ , i.e., iff  $h=22$

3. Find  $h$  such that  $\mathbf{b} = \begin{bmatrix} 4 \\ 6 \\ h \end{bmatrix}$  is in the span of the vectors  $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ ,  $\begin{bmatrix} 2 \\ 5 \\ 5 \end{bmatrix}$  and  $\begin{bmatrix} 7 \\ 19 \\ 13 \end{bmatrix}$ .

We require weights  $x_1$ ,  $x_2$  &  $x_3$  st

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 5 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ 19 \\ 13 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ h \end{bmatrix}$$

(i.e., so  $\begin{bmatrix} 4 \\ 6 \\ h \end{bmatrix}$  is a L.C. of these three vectors).

This represents the same system of eqns as in #2, so the answer is the same:  $\mathbf{b}$  is in the span of these 3 vectors

$\iff$  (iff)

$$\boxed{h=22}$$

4a. What does it mean to say that a set  $S = \{v_1, v_2, \dots, v_p\}$  is linearly independent? (I want the definition).

The set  $S = \{\vec{v}_1, \dots, \vec{v}_p\}$  is L.I. iff

the only soln to  $x_1 \vec{v}_1 + x_2 \vec{v}_2 + \dots + x_p \vec{v}_p = \vec{0}$  is

$$x_1 = x_2 = x_3 = \dots = x_p = 0$$

(i.e. all scalars must be taken to be 0)

4b. Give an example of three vectors in  $\mathbf{R}^2$  which form a linearly independent set, or explain why this is impossible.

It's impossible. No matter the choice of  $\vec{v}_1 = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ ,  $\vec{v}_2 = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$  and  $\vec{v}_3 = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ ,

there will be infinitely many solutions to  $x_1 \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + x_2 \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + x_3 \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ,

because, for example,  $\left[ \begin{array}{ccc|c} a_1 & b_1 & c_1 & 0 \\ a_2 & b_2 & c_2 & 0 \end{array} \right]$

must have at least one free variable.

