MATH 205A • Fall 2005

NAME

ROSS — suggested soln's.

MINI•EXAM 9/23/2005

Do NOT write here

1	
2	
3	
4	
TOTAL	

① show ALL your work!

② READ the questions!

③ BE NEAT!

GOOD LUCK!

1. Suppose the augmented matrix corresponding to some matrix equation $A\mathbf{x} = \mathbf{b}$ reduces to the following:

$$\left[\begin{array}{cccccccc|c} 1 & 2 & 0 & 0 & 5 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 & 4 & 0 & 0 & t \\ 0 & 0 & 0 & 1 & -3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r \end{array}\right].$$

1a. What (if any) conditions are there on r, s, and t so that ...

i. there are no solutions to $A\mathbf{x} = \mathbf{b}$.

There will be no solutions iff $r \neq 0$; the values of s & t do not matter.

ii. there is exactly one solution to $A\mathbf{x} = \mathbf{b}$.

There can never be exactly one. The existence of free variables means there are either NO soln's (if the system is inconsistent), or infinitely many soln's.

iii. there are infinitely many solutions to $A\mathbf{x} = \mathbf{b}$.

There are infinitely many iff $r = 0$; the values of r and s can be any real numbers (no restrictions)

1b. Write the solution to this system (given that there are solutions) in the form $\mathbf{v}_h + \mathbf{p}$ as done in class, that is, where $\mathbf{v}_h$ represents all solutions of the homogeneous system $A\mathbf{x} = \mathbf{0}$ and $\mathbf{p}$ is a particular solution to $A\mathbf{x} = \mathbf{b}$.

We have
$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -2x_2 - 5x_5 + 10 \\ x_2 \\ -4x_5 + t \\ 3x_5 + 8 \\ x_5 \\ x_6 \\ s \end{bmatrix} = \underbrace{x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -5 \\ 0 \\ -4 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}} + \begin{bmatrix} 10 \\ 0 \\ t \\ 8 \\ 0 \\ 0 \\ s \end{bmatrix}$$

where x_2, x_5, and x_6 are free.

(note $\vec{v}_h + \vec{p}$)

2. Consider the system of equations

$$\begin{aligned} x_1 + 2x_2 + 7x_3 &= 4 \\ 2x_1 + 5x_2 + 19x_3 &= 6 \\ 4x_1 + 5x_2 + 13x_3 &= h \end{aligned}$$

What (if any) conditions on h guarantee a solution exists? Show all your work.

The augmented matrix corresponding to this system is $\left[\begin{array}{ccc|c} 1 & 2 & 7 & 4 \\ 2 & 5 & 19 & 6 \\ 4 & 5 & 13 & h \end{array}\right]$

careful row reduction shows it equivalent to $\left[\begin{array}{ccc|c} 1 & 2 & 7 & 4 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 0 & h-22 \end{array}\right]$,

which has a sol'n (infinitely many, in fact) iff $h-22=0$, i.e., iff $\underline{\underline{h=22}}$

3. Find h such that $\mathbf{b} = \begin{bmatrix} 4 \\ 6 \\ h \end{bmatrix}$ is in the span of the vectors $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 5 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 19 \\ 13 \end{bmatrix}$.

We require weights x_1 & x_2 & x_3 ST

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 5 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ 19 \\ 13 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ h \end{bmatrix}$$

(ie, so $\begin{bmatrix} 4 \\ 6 \\ h \end{bmatrix}$ is a L.C. of these three vectors).

This represents the same system of eq'ns as in #3, so the answer is the same: $\vec{b}$ is in the span of these 3 vectors

$\Longleftrightarrow$ (iff)

$\boxed{h=22}$

4a. What does it mean to say that a set $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ is *linearly independent*? (I want the definition).

The set $S = \{\vec{v}_1, \ldots, \vec{v}_p\}$ is L.I. iff

the only sol'n to $x_1\vec{v}_1 + x_2\vec{v}_2 + \cdots + x_p\vec{v}_p = \vec{0}$ is

$$x_1 = x_2 = x_3 = \cdots = x_p = 0$$

(i.e. all scalars must be taken to be 0)

4b. Give an example of three vectors in $\mathbf{R}^2$ which form a linearly independent set, or explain why this is impossible.

It's impossible. No matter the choice of $\vec{v}_1 = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ and $\vec{v}_3 = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$,

there will be infinitely many solutions to $x_1\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + x_2\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + x_3\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$,

because, for example, $\left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & 0 \\ a_2 & b_2 & c_2 & 0 \end{array}\right]$

must have at least one free variable.