

1. Suppose the augmented matrix corresponding to some matrix equation  $A\mathbf{x} = \mathbf{b}$  reduces to the following:

$$\left[ \begin{array}{cccc|ccc} 1 & 2 & 0 & 0 & 5 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 & 4 & 0 & 0 & t \\ 0 & 0 & 0 & 1 & -3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r \end{array} \right].$$

1a. What (if any) conditions are there on  $r$ ,  $s$ , and  $t$  so that ...

i. there are no solutions to  $A\mathbf{x} = \mathbf{b}$ .

ii. there is exactly one solution to  $A\mathbf{x} = \mathbf{b}$ .

iii. there are infinitely many solutions to  $A\mathbf{x} = \mathbf{b}$ .

1b. Write the solution to this system (given that there are solutions) in the form  $\mathbf{v}_h + \mathbf{p}$  as done in class, that is, where  $\mathbf{v}_h$  represents all solutions of the homogeneous system  $A\mathbf{x} = \mathbf{0}$  and  $\mathbf{p}$  is a particular solution to  $A\mathbf{x} = \mathbf{b}$ .

2. Consider the system of equations

$$x_1 + 2x_2 + 7x_3 = 4$$

$$2x_1 + 5x_2 + 19x_3 = 6$$

$$4x_1 + 5x_2 + 13x_3 = h$$

What (if any) conditions on  $h$  guarantee a solution exists? Show all your work.

3. Find  $h$  such that  $\mathbf{b} = \begin{bmatrix} 4 \\ 6 \\ h \end{bmatrix}$  is in the span of the vectors  $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ ,  $\begin{bmatrix} 2 \\ 5 \\ 5 \end{bmatrix}$  and  $\begin{bmatrix} 7 \\ 19 \\ 13 \end{bmatrix}$ .

**4a.** What does it mean to say that a set  $S = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p\}$  is *linearly independent*? (I want the definition).

**4b.** Give an example of three vectors in  $\mathbf{R}^2$  which form a linearly independent set, or explain why this is impossible.