1. Suppose the augmented matrix corresponding to some matrix equation $A\mathbf{x} = \mathbf{b}$ reduces to the following:

$$\left[\begin{array}{ccccccc|c} 1 & 2 & 0 & 0 & 5 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 & 4 & 0 & 0 & t \\ 0 & 0 & 0 & 1 & -3 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r \end{array}\right].$$

1a. What (if any) conditions are there on r, s, and t so that ...

i. there are no solutions to $A\mathbf{x} = \mathbf{b}$.

ii. there is exactly one solution to $A\mathbf{x} = \mathbf{b}$.

iii. there are infinitely many solutions to $A\mathbf{x} = \mathbf{b}$.

1b. Write the solution to this system (given that there are solutions) in the form $\mathbf{v}_h + \mathbf{p}$ as done in class, that is, where $\mathbf{v}_h$ represents all solutions of the homogeneous system $A\mathbf{x} = \mathbf{0}$ and $\mathbf{p}$ is a particular solution to $A\mathbf{x} = \mathbf{b}$.

2. Consider the system of equations

$$\begin{aligned} x_1 + 2x_2 + 7x_3 &= 4 \\ 2x_1 + 5x_2 + 19x_3 &= 6 \\ 4x_1 + 5x_2 + 13x_3 &= h \end{aligned}$$

What (if any) conditions on h guarantee a solution exists? Show all your work.

3. Find h such that $\mathbf{b} = \begin{bmatrix} 4 \\ 6 \\ h \end{bmatrix}$ is in the span of the vectors $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 5 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 19 \\ 13 \end{bmatrix}$.

4a. What does it mean to say that a set $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ is *linearly independent*? (I want the definition).

4b. Give an example of three vectors in $\mathbf{R}^2$ which form a linearly independent set, or explain why this is impossible.