 NAME *Suggested solutions:*

1. Find $\begin{bmatrix} 1 & 3 & -5 & 6 \\ 0 & 1 & -3 & 2 \\ 2 & 5 & -7 & 10 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \\ 2 \end{bmatrix}$. $= 3\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 1\begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix} + 0\begin{bmatrix} -5 \\ -3 \\ -7 \end{bmatrix} + 2\begin{bmatrix} 6 \\ 2 \\ 10 \end{bmatrix}$

$$= \begin{bmatrix} 3+3+0+12 \\ 0+1+0+4 \\ 6+5+0+20 \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 31 \end{bmatrix}$$

2. Let A be the 3×4 matrix from problem 1. Express all solutions of $A\mathbf{x} = \mathbf{b}$ in the form $\mathbf{p} + \mathbf{v}_h$, where $\mathbf{p}$ is a particular solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{v}_h$ represents all solutions to the corresponding homogeneous equation $A\mathbf{x} = \mathbf{0}$. Here, $\mathbf{b} = \begin{bmatrix} 70 \\ 20 \\ 120 \end{bmatrix}$.

Hint. $\left[\begin{array}{cccc|c} 1 & 0 & 4 & 0 & 10 \\ 0 & 1 & -3 & 2 & 20 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ is row equivalent to the augmented matrix you would set up to begin solving this problem.

the row-equivalent matrix tells us the solutions of $A\vec{x} = \vec{b}$ (ie, $\begin{bmatrix} 1 & 3 & -5 & 6 \\ 0 & 1 & -3 & 2 \\ 2 & 5 & -7 & 10 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 70 \\ 20 \\ 120 \end{bmatrix}$)

are $\begin{cases} x_1 = 10 - 4x_3 \\ x_2 = 20 + 3x_3 - 2x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \end{cases} = \underbrace{\begin{bmatrix} 10 \\ 20 \\ 0 \\ 0 \end{bmatrix}}_{\vec{p}} + \underbrace{x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -2 \\ 0 \\ 1 \end{bmatrix}}_{\vec{v}_h}$

NOTE: it may help to write this: $\begin{cases} x_1 = 10 - 4x_3 + 0x_4 \\ x_2 = 20 + 3x_3 - 2x_4 \\ x_3 = 0 + 1x_3 - 0x_4 \\ x_4 = 0 + 0x_3 + 1x_4 \end{cases}$ to see this

3. Let A and $\mathbf{b}$ be as in problem 2. Let $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ and $\mathbf{a}_4$ be the columns of A. Use your answer to (2) to express $\mathbf{b}$ as a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ and $\mathbf{a}_4$ in 3 different ways, the first of which should be by setting all free variables to 0. Make clear your choices of values for the free variables. (See on-board example).

LC 1: we're told to use $x_3 = x_4 = 0$. This gives $x_1 = 10$ and $x_2 = 20$. Our L.C. is

$\boxed{10\vec{a}_1 + 20\vec{a}_2}$ (or, $10\vec{a}_1 + 20\vec{a}_2 + 0\vec{a}_3 + 0\vec{a}_4$)

LC 2:

for LC2 & LC3, there are ∞-many possible answers. Commonly seen on students' quizzes were these examples:

LC 3:

① choose $x_3 = 1$ and $x_4 = 0$. Get $6\vec{a}_1 + 23\vec{a}_2 + \vec{a}_3$

② $x_3 = 1$ and $x_4 = 1$. Get $6\vec{a}_1 + 21\vec{a}_2 + \vec{a}_3 + \vec{a}_4$

③ $x_3 = 2$ and $x_4 = 2$. Get $2\vec{a}_1 + 22\vec{a}_2 + 2\vec{a}_3 + 2\vec{a}_4$

etc. (so $x_1 = 10 - 8 = 2$ and $x_2 = 20 + (2 \cdot 3) + (2 \cdot -2) = 20 + 6 - 4 = 22$)