

1. Find $\begin{bmatrix} 1 & 3 & -5 & 6 \\ 0 & 1 & -3 & 2 \\ 2 & 5 & -7 & 10 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \\ 2 \end{bmatrix}$.

2. Let A be the 3×4 matrix from problem 1. Express all solutions of $A\mathbf{x} = \mathbf{b}$ in the form $\mathbf{p} + \mathbf{v}_h$, where \mathbf{p} is a particular solution of $A\mathbf{x} = \mathbf{b}$ and \mathbf{v}_h represents all solutions to the corresponding homogeneous equation $A\mathbf{x} = \mathbf{0}$. Here, $\mathbf{b} = \begin{bmatrix} 70 \\ 20 \\ 120 \end{bmatrix}$.

Hint. $\left[\begin{array}{cccc|c} 1 & 0 & 4 & 0 & 10 \\ 0 & 1 & -3 & 2 & 20 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$ is row equivalent to the augmented matrix you would set up to begin solving this problem.

3. Let A and \mathbf{b} be as in problem 2. Let $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ and \mathbf{a}_4 be the columns of A . Use your answer to (2) to express \mathbf{b} as a linear combination of $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ and \mathbf{a}_4 in 3 different ways, the first of which should be by setting all free variables to 0. Make clear your choices of values for the free variables. (See on-board example).

LC 1:

LC 2:

LC 3: