Math 205A Quiz 02 page 1 September 19, 2008 NAME SUGGESTED SOL'N'S.

1. Let $A = \begin{bmatrix} 5 & 2 & -5 & 12 \\ 6 & 1 & -13 & 13 \\ -2 & 0 & 6 & -4 \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

1A. Use the "super-augmented" matrix technique discussed in class to find what conditions (if any) there are on b_1, b_2, and b_3 which guarantee that $A\mathbf{x} = \mathbf{b}$ has a solution.

By TI-83, the RREF of $\left[\begin{array}{cccc|ccc} 5 & 2 & -5 & 12 & 1 & 0 & 0 \\ 6 & 1 & -13 & 13 & 0 & 1 & 0 \\ -2 & 0 & 6 & -4 & 0 & 0 & 1 \end{array}\right]$

is $\left[\begin{array}{cccc|ccc} 1 & 0 & -3 & 2 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 5 & 1 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 & -2 & -\frac{7}{2} \end{array}\right]$

(note: these two rows say when there IS a solution, that it's $\begin{cases} x_1 = (-\frac{1}{2}b_3) + 3x_3 - 2x_4 \\ x_2 = (1 \cdot b_2 + 3b_3) - 5x_3 - x_4 \end{cases}$ and x_3 & x_4 are free)

the last row tells us $A\vec{x} = \vec{b}$ has a sol'n $\Leftrightarrow$ $0 = b_1 - 2b_2 - \frac{7}{2}b_3$

1B. What value must b_1 have to ensure that $\mathbf{w} = \begin{bmatrix} b_1 \\ -13 \\ 12 \end{bmatrix}$ can be written as a linear combination of the columns of A?

from 1A we need

$0 = b_1 - 2(-13) - \frac{7}{2}(12)$; $0 = b_1 + 26 - 42$; $b_1 = 16$

1C. Express $\mathbf{w}$ from (1B) as a specific linear combination of the columns $\mathbf{a}_1$, $\mathbf{a}_2$, *etc.* of A. Your answer will have the form "$\mathbf{w} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \ldots$" (etc) where you'll supply the actual values of x_1, x_2, *etc.*.

from the RREF in 1A we have:

$\begin{cases} x_1 = (-\frac{1}{2} \cdot 12) + 3x_3 - 2x_4 \\ x_2 = (1 \cdot {-13} + 3 \cdot 12) + {-5}x_3 - x_4 \end{cases}$

where x_3 & x_4 are free; so

$\begin{cases} x_1 = -6 + 3x_3 - 2x_4 \\ x_2 = 23 - 5x_3 - x_4 \end{cases}$

we're asked for a SPECIFIC L.C.; it's easiest to let $x_3 = x_4 = 0$, so $x_1 = -6$ and $x_2 = 23$.

we get $\begin{bmatrix} 16 \\ -13 \\ 12 \end{bmatrix} = -6\begin{bmatrix} 5 \\ 6 \\ -2 \end{bmatrix} + 23\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + 0\begin{bmatrix} -5 \\ -13 \\ 6 \end{bmatrix} + 0\begin{bmatrix} 12 \\ 13 \\ -4 \end{bmatrix}$

other choices of x_3 & x_4 are allowed of course; for example if $x_3 = x_4 = 1$ then $x_1 = -5$ & $x_2 = 17$.

1D. Let $\mathbf{z} = \begin{bmatrix} 43 \\ 11 \\ 6 \end{bmatrix}$. Express all solutions of $A\mathbf{x} = \mathbf{z}$ in the form $\mathbf{p} + \mathbf{v}_h$ where $\mathbf{p}$ is a particular solution of $A\mathbf{x} = \mathbf{z}$ and $\mathbf{v}_h$ represents all solutions of $A\mathbf{x} = \mathbf{0}$.

again from the RREF in 1A (or, putting $\left[A \middle| \begin{smallmatrix} 43 \\ 11 \\ 6 \end{smallmatrix}\right]$ in "from scratch"),

we have $\begin{cases} x_1 = (-\frac{1}{2} \cdot 6) + 3x_3 - 2x_4 \\ x_2 = (1 \cdot 11 + 3 \cdot 6) - 5x_3 - x_4 \\ x_3 \text{ \& } x_4 \text{ are free} \end{cases}$

so $\begin{cases} x_1 = -3 + 3x_3 - 2x_4 \\ x_2 = 29 - 5x_3 - x_4 \\ x_3 = 1 \cdot x_3 \\ x_4 = 1 \cdot x_4 \end{cases}$

ie: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \overbrace{\begin{bmatrix} -3 \\ 29 \\ 0 \\ 0 \end{bmatrix}}^{\vec{p}} + \overbrace{x_3\begin{bmatrix} 3 \\ -5 \\ 1 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}}^{\vec{v}_h}$ where x_3 & x_4 are free!

1E. Explicitly give the "trivial solution" of $A\mathbf{x} = \mathbf{0}$.

simply $\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$