1. Let $A = \begin{bmatrix} 5 & 2 & -5 & 12 \\ 6 & 1 & -13 & 13 \\ -2 & 0 & 6 & -4 \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

1A. Use the "super-augmented" matrix technique discussed in class to find what conditions (if any) there are on b_1, b_2, and b_3 which guarantee that $A\mathbf{x} = \mathbf{b}$ has a solution.

1B. What value must b_1 have to ensure that $\mathbf{w} = \begin{bmatrix} b_1 \\ -13 \\ 12 \end{bmatrix}$ can be written as a linear combination of the columns of A?

1C. Express $\mathbf{w}$ from (1B) as a specific linear combination of the columns $\mathbf{a}_1$, $\mathbf{a}_2$, *etc.* of A. Your answer will have the form "$\mathbf{w} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \ldots$" (etc) where you'll supply the actual values of x_1, x_2, *etc.*.

1D. Let $\mathbf{z} = \begin{bmatrix} 43 \\ 11 \\ 6 \end{bmatrix}$. Express all solutions of $A\mathbf{x} = \mathbf{z}$ in the form $\mathbf{p} + \mathbf{v}_h$ where $\mathbf{p}$ is a particular solution of $A\mathbf{x} = \mathbf{z}$ and $\mathbf{v}_h$ represents all solutions of $A\mathbf{x} = \mathbf{0}$.

1E. Explicitly give the "trivial solution" of $A\mathbf{x} = \mathbf{0}$.