

1. Consider the augmented matrix $A = \left[\begin{array}{cc|c} 6 & 13 & 8 \\ 10 & 20 & 30 \end{array} \right]$.

1A. What system of linear equations is represented by A ?

$$\begin{cases} 6x_1 + 13x_2 = 8 \\ 10x_1 + 20x_2 = 30 \end{cases}$$

1B. Find the reduced row echelon form (rref) of A by hand, showing all the matrices produced along the way. Say what each elementary row operation was used to produce each new matrix from the previous one.

There are many "pathways" all leading to the same final rref of A ; here's but one:

$$\begin{aligned} \left[\begin{array}{cc|c} 6 & 13 & 8 \\ 10 & 20 & 30 \end{array} \right] &\xrightarrow{r_2 \leftarrow r_2/10} \left[\begin{array}{cc|c} 6 & 13 & 8 \\ 1 & 2 & 3 \end{array} \right] \xrightarrow{r_1 \leftarrow (r_1 - 6r_2)} \left[\begin{array}{cc|c} 0 & 1 & -10 \\ 1 & 2 & 3 \end{array} \right] \\ &\xrightarrow{\text{Swap } r_1, r_2} \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & -10 \end{array} \right] \xrightarrow{r_1 \leftarrow (r_1 - 2r_2)} \left[\begin{array}{cc|c} 1 & 0 & 23 \\ 0 & 1 & -10 \end{array} \right] \end{aligned}$$

← this is in rref.

1C. What is the solution of the system in part 1A?

$$\begin{cases} x_1 = 23 \\ x_2 = -10 \end{cases} \quad (\text{or as a "2-tuple", } (23, -10))$$

2. Consider the system of equations whose augmented matrix is $B = \left[\begin{array}{ccc|c} 2 & 1 & 13 & 12 \\ 1 & 0 & 5 & 7 \\ 3 & 4 & 27 & 12 \end{array} \right]$.

Find all the solutions of the system using the methods and notation we've developed in class. If there are no solutions, explain why. Otherwise, give a specific solution and show it satisfies the first equation the augmented matrix represents. (Use your calculator to find $\text{rref}(B)$).

$\text{rref}(B)$ via calculator is $\left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$ The last row of this matrix represents the equation $0x_1 + 0x_2 + 0x_3 = 1$, which has no solution. Thus the original system of equations represented by B is inconsistent.

3. Now consider the system of equations whose augmented matrix is $C = \left[\begin{array}{cccc|c} 2 & 1 & 13 & 12 & 10 \\ 1 & 0 & 5 & 7 & 6 \\ 3 & 4 & 27 & 12 & 14 \end{array} \right]$.

Again, find all the solutions of the system using the methods and notation we've developed in class. If there are no solutions, explain why; otherwise, give a specific solution and show it satisfies the first equation the augmented matrix represents. (Use your calculator to find the rref).

$\text{rref}(C)$ is $\left[\begin{array}{cccc|c} 1 & 0 & 5 & 0 & 34 \\ 0 & 1 & 3 & 0 & -10 \\ 0 & 0 & 0 & 1 & -4 \end{array} \right] \Rightarrow$ the solutions of the system represented by C are:

$$\begin{cases} x_1 = 34 - 5x_3 \\ x_2 = -10 - 3x_3 \\ x_3 \text{ is free} \\ x_4 = -4 \end{cases}$$

Now, one specific solution is obtained by choosing $x_3 = 1$; in this case $\begin{cases} x_1 = 29 \\ x_2 = -13 \\ x_3 = 1 \\ x_4 = -4 \end{cases}$

In the 1st equation we'd have: $2 \cdot 29 + 1 \cdot (-13) + 13(1) + 12(-4)$
 which is $58 - 13 + 13 - 48 = 10$ as required...