

1. Suppose $\mathbf{u}_1 = \begin{bmatrix} 1 \\ -2 \\ 5 \\ 4 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -2 \\ 3 \\ -13 \\ -8 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} -6 \\ 9 \\ -39 \\ -24 \end{bmatrix}$, and $\mathbf{u}_4 = \begin{bmatrix} 1 \\ -3 \\ 3 \\ 2 \end{bmatrix}$.

1A. What is the (initial, non-row reduced) augmented matrix which corresponds to the system of equations you need to solve in order to find out if the vector $\mathbf{b} = \begin{bmatrix} 15 \\ -31 \\ 77 \\ 50 \end{bmatrix}$ in the span of the vectors \mathbf{u}_1 , \mathbf{u}_2 , \mathbf{u}_3 , and \mathbf{u}_4 ?

1B. An augmented matrix which is row equivalent to the correct answer to (1A) is given below; you do *not* need to verify this. Use it to determine if \mathbf{b} is indeed in the span of \mathbf{u}_1 , \mathbf{u}_2 , \mathbf{u}_3 , and \mathbf{u}_4 . If \mathbf{b} is not in the span, explain why not. If it is in the span, find all the linear combinations of \mathbf{u}_1 , \mathbf{u}_2 , \mathbf{u}_3 , and \mathbf{u}_4 which add up to \mathbf{b} (that is, find the solution of the system of equations referred to in (1A); if there are any solutions write them in terms of the free variables (if there are any)).

Row equivalent matrix: $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 3 & 0 & -4 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$.

1C. If the vector \mathbf{b} had been $\begin{bmatrix} 15 \\ -31 \\ 77 \\ 57 \end{bmatrix}$ instead, the corresponding augmented matrix would've been different as shown below. Would this new \mathbf{b} be in the span of \mathbf{u}_1 , \mathbf{u}_2 , \mathbf{u}_3 , and \mathbf{u}_4 ? Explain why or why not.

New row equivalent augmented matrix: $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 3 & 0 & -4 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 7 \end{array} \right].$

1D. Is there a vector \mathbf{b} in the span of \mathbf{u}_1 , \mathbf{u}_2 , \mathbf{u}_3 , and \mathbf{u}_4 which can only be written as a linear combination of these four vectors in *exactly one way*? Or if there's at least one way, will there be infinitely many? Explain. (Hint: no matter what \mathbf{b} you attempt to write as a LC of \mathbf{u}_1 , \mathbf{u}_2 , \mathbf{u}_3 , and \mathbf{u}_4 , will there be any difference in the first four columns of the reduced echelon form of the resulting augmented matrix? Look at the first four columns of those given in (1B) and (1C))