Math 205A Quiz 01 page 1 September 12, 2008 NAME *Suggested Sol'n*

1. Consider the system of equations

$$\begin{aligned} x_3 + 3x_4 &= p \\ x_1 + 5x_2 + 3x_3 + 7x_4 &= t \\ 20x_1 + 100x_2 + 62x_3 + 146x_4 &= w, \end{aligned}$$

where p, t, and w are three real numbers whose values we don't know.

1A. What is the augmented matrix for this system?

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & 3 & p \\ 1 & 5 & 3 & 7 & t \\ 20 & 100 & 62 & 146 & w \end{array}\right]$$

1B. By hand, find the matrix in RREF which is row equivalent to the answer in (1A). Show and label all your steps as we've done in class, for example, if you add 4 copies of row 5 to row 8, write "$r_8 \leftarrow r_8 + 4r_5$"; if you swap rows 4 and 5, write "swap r_4 and r_5", etc. (you don't need the quotes) Use steps that make the work easy; avoid fractions if possible. Note well that your column with p, t, and w will change as you change the coefficient side of the matrix.

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & 3 & p \\ 1 & 5 & 3 & 7 & t \\ 20 & 100 & 62 & 146 & w \end{array}\right] \xrightarrow{r_3 \leftarrow r_3 - 20r_2} \left[\begin{array}{cccc|c} 0 & 0 & 1 & 3 & p \\ 1 & 5 & 3 & 7 & t \\ 0 & 0 & 2 & 6 & w-20t \end{array}\right] \xrightarrow{r_3 \leftarrow r_3 - 2r_1}$$

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & 3 & p \\ 1 & 5 & 3 & 7 & t \\ 0 & 0 & 0 & 0 & w-20t-2p \end{array}\right] \xrightarrow[r_1 \,\&\, r_2]{\text{swap}} \left[\begin{array}{cccc|c} 1 & 5 & 3 & 7 & t \\ 0 & 0 & 1 & 3 & p \\ 0 & 0 & 0 & 0 & w-20t-2p \end{array}\right]$$

$$\xrightarrow{r_1 \leftarrow r_1 - 3r_2} \left[\begin{array}{cccc|c} 1 & 5 & 0 & -2 & t-3p \\ 0 & 0 & 1 & 3 & p \\ 0 & 0 & 0 & 0 & w-20t-2p \end{array}\right]$$

I accepted this answer HOWEVER, if $w-20t-2p \neq 0$ then to be in RREF this last entry must be 1 and the entries above it must be 0's:

1C. Use the answer to (1B) to determine what relationship p, t, and w must satisfy in order for this system to have a solution.

The original system (top of the page) is consistent

$$\Longleftrightarrow$$

$$\boxed{w - 20t - 2p = 0}$$

this requires 3 more steps:

$$\xrightarrow[\text{by } w-20t-2p]{\text{divide row 3}} \left[\begin{array}{c|c} (\text{same}) & \begin{matrix} t-3p \\ p \\ 1 \end{matrix} \end{array}\right] \xrightarrow[r_2 \leftarrow r_2 - p r_3]{r_1 \leftarrow r_1 - (t-3p)r_3} \left[\begin{array}{c|c} (\text{same}) & \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \end{array}\right] \text{ !!! } \ldots$$

Suggested solution

2. Evaluate the following: $\begin{bmatrix} 3 & 2 \\ 5 & -1 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$$\begin{bmatrix} 3 & 2 \\ 5 & -1 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} = 4\begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix} + 5\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 24 \end{bmatrix} + \begin{bmatrix} 10 \\ -5 \\ 20 \end{bmatrix} = \begin{bmatrix} 22 \\ 15 \\ 44 \end{bmatrix}$$

3. Let $A = \begin{bmatrix} 1 & 4 & 4 & 3 \\ 3 & 12 & 13 & 11 \\ 2 & 8 & 7 & 4 \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} 18 \\ 61 \\ 29 \end{bmatrix}$.

3A. Show how to write **b** as a linear combination of the columns of A (find all solutions and express them in terms of any free variables).

by calculator, the RREF of $[A|\vec{b}]$ is $\left[\begin{array}{cccc|c} 1 & 4 & 0 & -5 & -10 \\ 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

which says: $\vec{b} = x_1\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} 4 \\ 12 \\ 8 \end{bmatrix} + x_3\begin{bmatrix} 4 \\ 13 \\ 7 \end{bmatrix} + x_4\begin{bmatrix} 3 \\ 11 \\ 4 \end{bmatrix}$ ← (N.B. here is the L.C. & this needs to be part of your answer)

where $\begin{cases} x_1 = -10 - 4x_2 + 5x_4 \\ x_3 = 7 - 2x_4 \\ \text{and } x_2 \text{ and } x_4 \text{ are free} \end{cases}$

3B. Do the columns of A span $\mathbf{R}^3$? Explain your answer.

NO. There are $\vec{b}$'s for which the last row of the RREF of $[A|\vec{b}]$ will be $[0\,0\,0\,0|1]$ instead of all 0's, i.e. the last row will lead to an inconsistency in solving $A\vec{x} = \vec{b}$. Such $\vec{b}$'s will not be in the span...

ALTERNATIVELY: Not every row of A has a pivot position so by theorem 4, A's columns cannot span $\mathbb{R}^3$.