

$$\begin{aligned} & x_3 + 3x_4 = p \\ \text{1. Consider the system of equations} & \quad x_1 + 5x_2 + 3x_3 + 7x_4 = t \\ & 20x_1 + 100x_2 + 62x_3 + 146x_4 = w, \end{aligned}$$

where p , t , and w are three real numbers whose values we don't know.

1A. What is the augmented matrix for this system?

1B. By hand, find the matrix in RREF which is row equivalent to the answer in (1A). Show and label all your steps as we've done in class, for example, if you add 4 copies of row 5 to row 8, write " $r_8 \leftarrow r_8 + 4r_5$ "; if you swap rows 4 and 5, write "swap r_4 and r_5 ", etc. (you don't need the quotes) Use steps that make the work easy; avoid fractions if possible. Note well that your column with p , t , and w will change as you change the coefficient side of the matrix.

1C. Use the answer to (1B) to determine what relationship p , t , and w must satisfy in order for this system to have a solution.

2. Evaluate the following:

$$\begin{bmatrix} 3 & 2 \\ 5 & -1 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

3. Let $A = \begin{bmatrix} 1 & 4 & 4 & 3 \\ 3 & 12 & 13 & 11 \\ 2 & 8 & 7 & 4 \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} 18 \\ 61 \\ 29 \end{bmatrix}$.

3A. Show how to write \mathbf{b} as a linear combination of the columns of A (find all solutions and express them in terms of any free variables).

3B. Do the columns of A span \mathbf{R}^3 ? Explain your answer.