

**MATH106A,B CALCULUS II - PROF. P. WONG**

FINAL EXAM - APRIL 16, 2009

**NAME:**

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

*Advice:* DON'T spend too much time on a single problem.

<b>Problems</b>	<b>Maximum Score</b>	<b>Your Score</b>
1.	21	
2.	18	
3.	18	
4.	17	
5.	21	
6.	21	
7.	18	
8.	16	
<b>Total</b>	<b>150</b>	

1. Evaluate each of the following integrals.

(7 pts.)(a)

$$\int_0^1 \frac{\arctan x}{1+x^2} dx$$

(7 pts.)(b)

$$\int \sqrt{y} \ln y \, dy$$

(7 pts.)(c)

$$\int \frac{6}{z^2 + z - 2} dz$$

2. (9 pts.)(a) Evaluate the improper integral

$$\int_0^2 \frac{1}{\sqrt{4-x^2}} dx$$

(9 pts.)(b) Find the indefinite integral

$$\int \frac{x^3 + x + 1}{1 + x^2} dx$$

3. Consider the following Initial Value Problem.

$$y' = \frac{\sqrt{y}}{x}, \quad y(1) = 1.$$

(9 pts.)(a) Use the method of separation of variables to solve this IVP.

(9 pts.)(b) Estimate the value  $y(2)$  (when  $x = 2$ ) of the solution using Euler's method with two steps with initial point  $(1, 1)$ . DO THIS BY HAND.

(17 pts.)**4.** Find the **exact** volume of the solid formed when the region bounded by the curves  $y = x^2$  and  $y = x^3$  is revolved around the vertical  $y$ -axis. [First sketch a picture of the region.]

5. Determine whether each of the following series converges or diverges. **Justify your answer.**

(7 pts.)(a)

$$\pi + \frac{1}{4} + \frac{1}{12} + \frac{1}{36} + \frac{1}{108} + \dots$$

(7 pts.)(b)

$$\sum_{n=1}^{\infty} n e^{-n}$$

(7 pts.)(c)

$$\sum_{k=0}^{\infty} \frac{1}{k + 106}$$

6. For each of the following series, determine whether the series converges absolutely, conditionally, or neither.

(7 pts.)(a)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^3}{4^n}$$

(7 pts.)(b)

$$\sum_{k=2}^{\infty} \frac{(-1)^k k^2}{k^3 - 2}$$

(7 pts.)(c)

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{2n + 1}$$

**7.**

(9 pts.)(a) Let  $f(x) = \frac{3x^2}{(1+x^3)^2}$ . Use the Maclaurin series for  $\frac{1}{1-t}$  to find the Maclaurin series for  $f(x)$ . [Hint: Is  $f(x)$  the derivative of some function?]

(9 pts.)(b) Find the interval of convergence for the following power series. [Don't forget to check the endpoints.]

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{2k}$$

**8.**

(8 pts.)(a) Find the second order Taylor polynomial for  $f(x) = (1 - x^2)e^x$  centered at  $x_0 = 1$ .

(8 pts.)(b) Consider the function

$$h(x) = \begin{cases} 1 - kx, & \text{if } 0 \leq x \leq \frac{1}{k}; \\ 0, & \text{otherwise.} \end{cases}$$

For what value of  $k$  is  $h(x)$  a probability density function? Justify your answer.