Math 106D

Calculus 2

Final Exam

April 16, 2009

Name__________________

Mr. Balcomb

1. Let I = $\int_0^1 x^2 dx$
(6 pts)

A. Use the Fundamental Theorem of Calculus to evaluate I exactly.

B. Compute the approximating sum L_4 . Show your work.

C. Compute the approximation error $|I - L_4|$

2. Find the integrals:
   (5 pts each)

   A. $\int \frac{3x}{1-x^2}dx$

   B. $\int \frac{x+1}{x^2-5x+6}dx$

   C. $\int \frac{sec^2(x)}{\sqrt{\tan(x)}}dx$

3. For f(x) = $\sqrt{x}$
(9 pts)

A. Give the second degree Taylor polynomial for f based $x_0 = 16$.

B. Use this polynomial to estimate $\sqrt{17}$.

C. What is the possible error that could have occurred in your estimate in part B? Recall that if you use the Taylor polynomial of degree n at x_o to approximate f(x) for x in an interval I containing x_o then $\frac{K_{n+1}|x - x_0|^{n+1}}{(n+1)!}$ is an upper bound for the approximation error. [K_{n+1} is an upper bound for the absolute value of the (n+1)st derivative of f on I .]

4. Describe how you would integrate $\int sin^m(x)cos^n(x)dx$ if m and n are both odd and positive integers. (5 pts)

5. Find the volume of the solid formed by revolving the region bounded by the graph of y = sin(x) and y = 0 in the interval $[0,\pi]$ about the x-axis. (5 pts)

6. Find the solution that passes through (0,2) for the equation $\frac{dy}{dx} = \frac{1}{(x^2+1)^{\frac{3}{2}}}$.
(5 pts)

7. Find the arc length of the graph y = ln(cos x) from x=0 to x= $\frac{\pi}{4}$.
(5 pts)

8. Do these integrals converge? Justify your answer. (5 pts each)

A. $\int_1^\infty xe^{-x}dx$.

B. $\int_3^\infty \frac{x}{(x+1)(x-2)}dx$.

9. For each of the following series, test to see whether it converges or diverges and explain why. (5 pts each)

A. $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$

B. $\sum_{k=1}^{\infty} \frac{k!}{e^k}$

C. $\sum_{n=2}^{\infty} \frac{(-1)^n \sqrt{n}}{n-1}$

10. For the series $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$,
(8 pts)

A. Give its radius of convergence:

B. Give its interval of convergence.

11. Starting with the Maclaurin series for sin(x),
(9 pts)

A. Find a power series expression for $x\sin(x)$.

B. Now find a power series expression for $\int x sin(x) dx$.

C. Using this formula, approximate $\int_0^{\pi} x sin(x) dx$ with an error less than 0.01. Justify your answer.

12. For the series $\sum_{1}^{\infty}\left[\frac{1}{n^2}-\frac{1}{2^n}\right]$,
(8 pts)

A. Show that it converges.

B. How large must n be to ensure that S_n differs from S by less than 0.01? Justify your answer.