LINEAR ALGEBRA FINAL EXAM April 16, 2004

Name: ______________________________

While the final answer is important, you earn points for all the work leading to that answer, as well as the answer itself. Show all your steps clearly so you will be eligible for the most partial credit. Good luck!

1.) (10 pts.) **True or False:** The set of all linear combinations of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ is a vector space. *If True:* explain why in detail. *If False:* explain why in detail, and/or provide a counterexample, that is, an example to show when the statement is false.

2.) (10 pts.) Compute the inverse of the matrix $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -3 \end{bmatrix}$ using the algorithm we learned.

3.) (10 pts.) Explain what is wrong with the following discussion: Let $\mathbf{f}(t) = 3 + t$ and $\mathbf{g}(t) = 3t + t^2$, and note that $\mathbf{g}(t) = t\mathbf{f}(t)$. Then $\{\mathbf{f}, \mathbf{g}\}$ is linearly dependent because $\mathbf{g}$ is a multiple of $\mathbf{f}$.

4.) (10 pts.) **True or False:** If a set contains more vectors than there are entries per vector, then the set is linearly independent. *If True:* explain why in detail. *If False:* explain why in detail, and/or provide a counterexample, that is, an example to show when the statement is false.

5.) (10 pts.) If the null space of a 5×6 matrix A is 4-dimensional,

a.) what is the dimension of the row space of A?

b.) and the column space of A is a subspace of $\mathbb{R}^k$, what is k?

c.) and the null space of A is a subspace of $\mathbb{R}^k$, what is k?

6.) (10 pts.) For $A = \begin{bmatrix} 1 & -2 & 0 & 4 & 0 \\ 0 & 0 & 1 & -9 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, find an explicit description of Nul A by listing vectors that span the null space.

7.) (10 pts.) Let $\mathcal{E} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis for $\mathbb{R}^3$, $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be a basis for a vector space V, and $T : \mathbb{R}^3 \to V$ be a linear transformation with the property that

$$T(x_1, x_2, x_3) = (x_3 - x_2)\mathbf{b}_1 - (x_1 + x_3)\mathbf{b}_2 + (x_1 - x_2)\mathbf{b}_3.$$

a.) Compute $T(\mathbf{e}_1)$, $T(\mathbf{e}_2)$, and $T(\mathbf{e}_3)$.

b.) Compute $[T(\mathbf{e}_1)]_{\mathcal{B}}$, $[T(\mathbf{e}_2)]_{\mathcal{B}}$, and $[T(\mathbf{e}_3)]_{\mathcal{B}}$.

c.) Find the matrix for T relative to $\mathcal{E}$ and $\mathcal{B}$.

8.) (10 pts.) Classify the quadratic form $2x_1^2 + 10x_1x_2 + 2x_2^2$ (positive definite, negative semidefinite, etc.). Then make a change of variable, $\mathbf{x} = P\mathbf{y}$, that transforms the quadratic form into one with no cross-product terms. Write the new quadratic form.

9.) (10 pts.) Given $A = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 3 \\ 2 & 5 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ -4 \\ 2 \end{bmatrix}$, find a least-squares solution of $A\mathbf{x} = \mathbf{b}$ by constructing the normal equations for $\hat{\mathbf{x}}$ and solving for $\hat{\mathbf{x}}$. Describe what is happening geometrically when we compute a least-squares solution.

10.) (10 pts.) Determine whether the columns of $\begin{bmatrix} 12 & 2 & 3 \\ 3 & -3 & 23 \\ -5 & 3 & 21 \end{bmatrix}$ are orthogonal. Is the matrix orthogonal?

BONUS: Check *one* of the following options.

________ OPTION 1: *For 5 points* - write a poem involving one or more topics we discussed this semester in Linear Algebra class. You may use the back of this page, or any other page where you have space. Just let me know where to look.

________ OPTION 2: *For a variable number of points* - if your grade on the final exam is higher than at least one of your earlier exam grades, your previous lowest exam grade will be replaced by a new grade: the average of your previous score with your final exam score.