NAME____

I___II___III___IV___V___VI___VII___VIII___ IX___ X___XI___ XII___ TOTAL____

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	TOTAL
(8)	(12)	(16)	(8)	(4)	(4)	(8)	(10)	(10)	(10)	(5)	(5)	(100)

April 15, 2009

Mathematics 206a
Multivariable Calculus
Final Examination

Mr. Haines

The volume of a right circular cylinder of height h and radius r is $V = \pi r^2 h$.

The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

The cylindrical coordinate conversion formulas are $x = r\cos\theta; y = r\sin\theta; z = w$.

(8) I. Give a parametrization for:

A. The line segment from the point (2, 4, 5) to the point (3, 6, 7) .

B. The plane rectangular surface in $\mathbb{R}^3$ with corners at the points (0, 2, 0), (4, 2, 0), (4, 2, 3), and (0, 2, 3).

(12) II. If $\mathbf{F}(x, y, z) = -y\mathbf{i} + x\mathbf{j} + 5\mathbf{k}$ and C is the curve parametrized by $\mathbf{c}(t) = (2\cos t, 2\sin t, 5)$ with $0 \le t \le 2\pi$.

A. Explain why C is or is not a closed curve.

B. Calculate the value of $\int_C \vec{F} \bullet d\vec{x}$

C. Explain why your answer to Part B proves that **curl F** is not the zero vector.

(16). III. The surface M in $\mathbb{R}^3$ is parametrized by $\mathbf{f}: \mathbb{R}^2 \to \mathbb{R}^3$ with rule $\mathbf{f}(s, t) = (s, 2t, st^2)$ with $0 \le s \le 2$ and $0 \le t \le 4$.

A. Set up but do not evaluate an integral that gives the surface area of M .

B. Compute the unit normal to M at the point (1, 4, 4).

C. Give an equation (either coordinate or parametric) for the tangent plane to M at the point (1, 4, 4).

D. Give a parametrization of any curve C that lies in the surface M and passes through the point (1, 4, 4) .

(8) IV. Give examples of

A. two vectors in $\mathbb{R}^3$ whose cross product is $\mathbf{i} + \mathbf{j} + \mathbf{k}$.

B. a vector field on $\Re^3$ with divergence $xy + 6x + 2$.

(4) V. Suppose $\boldsymbol{f}\,(x,y)=(x^3y,\ 2x^2+y)$. Write a formula for D***f***(1,1), the derivative of ***f*** at (1,1).

(4) VI. If $f:\Re^2 \rightarrow \Re$ has rule $f(x,y)=x^2+2xy+3y^2$, calculate the directional derivative of *f* at (1, 2) in the direction parallel to the vector 3**i** + 4**j**.

(8) VII. For the quadratic form p(x, y, z) = $x^2 + 3xz + z^2$

A. Give a symmetric matrix S that is the matrix of this quadratic form.

B. By taking determinants (and using Sylvester's Theorem), determine if p is positive definite, negative definite, indefinite, or none of these.

(10) VIII. Evaluate $\oiint_m \mathbf{F} \bullet \mathbf{n}\, d\sigma$, where $\mathbf{F}(x, y, z) = (y^2, z^2 + 2y, y^2)$ and M is the surface of the cylinder defined by $x^2 + y^2 \leq 9$ between the planes $z = 2$ and $z = 7$.

(10) IX. Given the vector field **F** (x, y, z) = (2, z^2, 2yz) and the path C in $\Re^3$ parametrized by **c** (t) = $\left(\frac{t^4}{2}, \sin\left(\frac{t\pi}{2}\right), t\right)$ with $0 \le t \le 2$, calculate the path integral $\int_C \mathbf{F} \bullet d\mathbf{x}$.

(10) X. Compute $\oint_C \mathbf{F} \bullet d\mathbf{x}$ where C is the counterclockwise-oriented boundary of the triangular region in the first quadrant bounded by the curves $x = 0$, $y = 0$, and $x + y = 1$

A. if **F** $(x, y) = (y^2, y + x^2)$.

B. if **F** $(x, y) = (2x^4 + 3y, 3x + 2y^5)$.

(5) XI. If S is the solid bounded by the surfaces $9 = x^2 + y^2$, $z = 2$, and $z = 7$, set up, but do <u>not</u> evaluate, the iterated integral that results from changing the triple integral $\iiint_S (x^2 + y^2 + z^2)\,dxdydz$ to cylindrical coordinates.

(5) XII. If $f : \Re^2 \to \Re$ with rule $f(x, y) = x^2 + 3xy - y^2$ calculate the Hessian of f at (1, 2).