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I____II____III____IV____V____VI____VII____VIII____IX___X___TOTAL ___________

April 15 2004

Mathematics 206a
Multivariable Calculus
Final Examination

Mr. Haines

I. (5) If $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ and L is the line with parametric equation $\mathbf{x} = (1, 2, 3) + t\mathbf{a}$ for all $t \in \Re$, give the equation of any line through the point $(1, 2, 3)$ that is perpendicular L

II.(10) The *curvature* of a curve C with parametrization $\mathbf{c}: \Re \to \Re^3$ is the number

$$\kappa = \frac{\|c'(t) \times c''(t)\|}{\|c'(t)\|^3} .$$

A. If C is any straight line, such as one of the lines in exercise I, calculate the curvature of C .

B. If C is the circle of radius 3 centered at the point (0, 0, 4) and lying in the plane z = 4, calculate the curvature of C.

III. (5) Suppose $f(x, y) = xy^2 + x^2 y + 5x$ and $\mathbf{a} = (1, 1)$ . What is the value of the directional derivative of $f$ at **a** in the direction parallel to the line y = 3x .

IV. (5) If $f(x, y) = x^4 + xy + y^4$, give the Hessian for $f$ at $(1, 0)$ .

V. (10) If $\mathbf{g}: \Re^2 \to \Re^3$ is the parametrization of the unbounded surface S in $\Re^3$ with formula $\mathbf{g}(s, t) = (st, s-t, s+t)$, and $\mathbf{a}$ is the point with $(s, t) = (2, 3)$:

A) Compute the Jacobian of **g** at **a**.

B) The total derivative of **g** at **a**, $(\mathbf{Dg}(\mathbf{a})): \Re^2 \to \Re^3$ is also a surface in $\Re^3$. Give the formula for $(\mathbf{Dg}(\mathbf{a}))(x, y)$. What kind of a transformation is it?

VI. (10) Let R be the triangle with vertices (0, 0), (0, 1), and (1, 1). Evaluate the double integral

$$\iint_R (x + y)\,dA$$

VII. (10) If M is the part of the surface whose equation is $z = x^2 - y^2$ that lies inside the cylinder whose equation is $x^2 + y^2 = 1$, give a parametrization for $\partial M$. M looks like a potato chip.

VIII. (15) Let **F** be a vector field given by $\mathbf{F}(x, y, z) = (ye^z, xe^z, xye^z)$.

Let C be the boundary of the square in the xy-plane with vertices (0, 0, 0), (1, 0, 0), (1, 1, 0), (0, 1, 0), (0, 0, 0) oriented in that order and let D be the diagonal of that square connecting (0, 0, 0) to (1, 1, 0).

A. Find a potential function for **F**.

B. Evaluate $\int_C \mathbf{F} \bullet d\mathbf{x}$ .

C. Evaluate $\int_D \mathbf{F} \bullet d\mathbf{x}$ .

IX. (15) Let $S = \{(x, y, z) \mid x^2 + y^2 + z^2 \le 4\}$ be the solid ball of radius 2 centered at the origin. Recall that the volume of a ball or radius $r$ is $(4/3)\pi r^3$ and that $\partial S = \{(x, y, z) \mid x^2 + y^2 + z^2 = 4\}$ is the spherical boundary of S .

A. If (a, b, c) is a point on the surface $\partial S$, give the equation of the tangent plane to $\partial S$ at the point (a, b, c). Your answer will contain two a's, two b's, and two c's because the tangent plane depends on the point (a, b, c).

B. Calculate **n**, the unit normal vector to the surface $\partial S$ at any point (x, y, z) . Your answer will contain x's, y's, and z's because the direction of the unit normal changes depending on where (x, y, z) lies on $\partial S$.

C. Find a vector field $\mathbf{F} = (F_1, F_2, F_3)$ on $\Re^3$ with the property that $\mathbf{F} \bullet \mathbf{n} = x^2 + y + z$ .

IX (cont.)

D. Calculate div **F** , the divergence of **F** .

E. The Divergence Theorem (Gauss' Theorem) says $\iiint_S div\mathbf{F}dV = \oiint_{\partial S} \mathbf{F} \bullet \mathbf{n} d\sigma$ for suitable S and **F**. Use this theorem and other information given above to calculate

$$\oiint_{\partial S} (x^2 + y + z) d\sigma$$

X. (15) Let M be the portion of the surface of the sphere with radius $\sqrt{5}$ and center (0, 0, -2) that is above the xy-plane and let **F** be a vector field given by **F**(x, y, z) = (y, -x, $e^{xz}$) .

A) What is the coordinate equation of this sphere (in terms of x, y, and z)?

B) What is the intersection of this sphere with the xy-plane?

C) Stokes's Theorem says $\iint_M curl\,\mathbf{F} \bullet \mathbf{n}\; d\sigma = \oint_{\partial M} \mathbf{F} \bullet dx$ for suitable M and **F**. Evaluate

$\iint_M curl\,\mathbf{F} \bullet \mathbf{n}\; d\sigma$ by first converting it to a line integral using Stokes's Theorem and calculating the resulting line integral.