Name:____

Math 206A: Winter 2012
Final Exam

Put your name on this exam sheet and turn it in with your exam book. Write all of your answers (except for 4b) in the exam book. Label problems clearly and circle final answers.

Correct answers accompanied by incorrect or incomplete work will not receive full credit.

Good Luck!

1. (12 points) For each of the following equations:

   (a) Identify the name of the surface.

   (b) Match it with a sketch (sketches may not be to scale and may not be oriented in the usual direction).

   (c) Find the coordinates of the center, vertex, or saddle point (when appropriate).

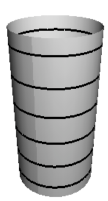
   (d) Describe the shape of the traces parallel to the xy-plane.

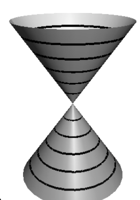
   (It is not necessary to show work for this question. But if you get the question wrong, some work might be worth partial credit.)

   (i) $5(x+11)^2 - 3y^2 - z = 0$

   (ii) $5x^2 + 3(y-7)^2 = 1$

   (iii) $5x^2 - y + 3z^2 = 0$


(U)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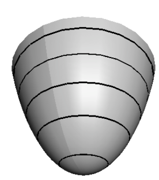
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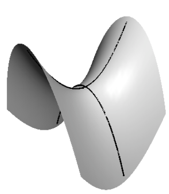
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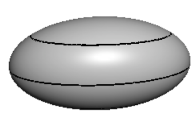
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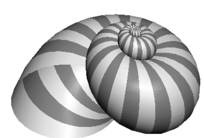

(Z)

2. (13 points) $\vec{l}(t) = (t-1, -2t+3, 5t)$ is the parametric equation of a line.

   (a) Find a point on the line.

   (b) Find a vector parallel to the line.

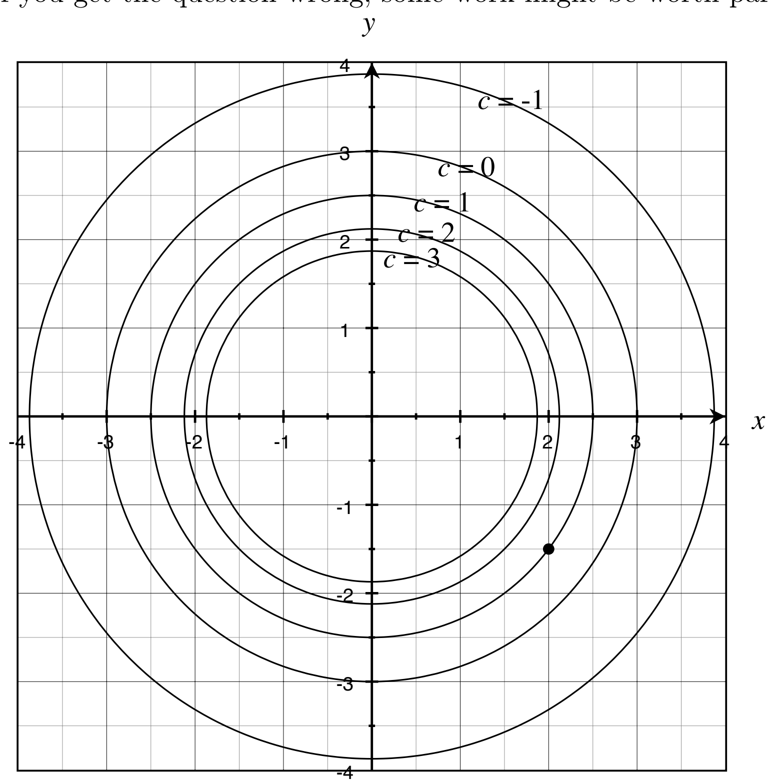
   (c) Write the equation of the plane that contains $\vec{l}(t)$ and the point $(3,1,4)$.
   Simplify to $Ax + By + Cz = D$ form.

3. (4 points) Let $f : \mathbb{R}^2 \to \mathbb{R}$ have the following properties.

- $f(2,3) = 5$
- $f(3,5) = 8$
- $\lim_{(x,y)\to(2,3)} f(x,y) = 8$
- $\lim_{(x,y)\to(3,5)} f(x,y) = 8$

Is f continuous at $(3,5)$? Why or why not?

4. (16 points) Consider the contour plot for the function $f(x,y)$. (It is not necessary to show work for this question. But if you get the question wrong, some work might be worth partial credit.)



(a) Determine whether the following quantities are *positive, negative,* or *zero.*

i. $\dfrac{\partial f}{\partial x}(2,-1.5)$

ii. $\dfrac{\partial^2 f}{\partial x^2}(2,-1.5)$

iii. $D_{\vec{u}} f(2,-1.5)$ where $\vec{u} = -3\,\hat{\imath} + \hat{\jmath}$.

(b) On the graph above sketch an approximation of $\vec{\nabla} f(2,-1.5)$.

5. (13 points) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = \sqrt[3]{3x^2 + xy^2 + 6y^3}$.

   (a) Calculate $Df(-1, 1)$.

   (b) Describe how the graphs of $L(x, y) = f(-1, 1) + Df(-1, 1)\big((x, y) - (-1, 1)\big)$ and $f(x, y)$ are related.

   (c) Why is it reasonable to write $L(-1.1, 1.1) \approx f(-1.1, 1.1)$, but it is not reasonable to write $L(-10, 10) \approx f(-10, 10)$?

6. (14 points) Determine whether the following statements are TRUE or FALSE (write out the words "true" or "false"). Briefly explain your choice.

   (a) The entries in the Hessian matrix of $f : \mathbb{R}^n \to \mathbb{R}$ are first order partial derivatives.

   (b) Consider a function $f : \mathbb{R}^2 \to \mathbb{R}$. If $\det(Hf(\vec{a})) > 0$, then $\vec{a}$ is a saddle point of f.

   (c) If $f(x, y) = 25x^2 - 20xy + 4y^2$, then $(4, 10)$ is a critical point.

7. (5 points) Consider a piece of wire that follows the curve C parameterized by

$$\vec{f}(t) = (3t^3 + 1t^2 + 4t, 1e^t - 5); \quad 0 \le t \le 3.$$

   The density of the wire varies according to the function

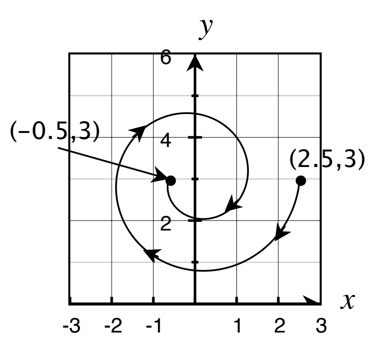
$$u(x, y) = x^2 + y + 20.$$

   Set up (but **do not evaluate**) the integral that represents the weight of the wire. Simplify so that there is no vector notation in your answer.

8. (9 points) Let $\vec{F} = \left(-e^{-x} \ln y, \dfrac{e^{-x}}{y}\right)$.

   (a) Show that $\vec{F}$ is path independent.

   (b) Use the fundamental theorem of path integrals to evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{x}$, where C is the curve shown.



9. (10 points) Let C be the unit circle, oriented counterclockwise and let $\vec{F} = (y, -x)$.

(a) Without using Green's theorem evaluate the line integral $\oint_C \vec{F} \cdot d\vec{x}$.

(b) Assume that the hypotheses of Green's Theorem are satisfied. Set up the iterated integral that you would use if you were using Green's Theorem to evaluate the line integral $\oint_C \vec{F} \cdot d\vec{x}$. (Do NOT evaluate.)

10. (4 points) What was your favorite topic this semester?