

Math 106: Review for Final Exam, Part I

1. Find the following. [See Review for Exam II for integration tips and strategies.]

(a) $\int 12x^2 \cos(x^3) dx$

(b) $\int_0^{\infty} xe^{-3x} dx$

(c) $\int_0^6 \frac{dx}{(x-4)^2}$

(d) $\int \frac{3x^2 + 2x - 5}{(x^2 + 1)(x - 4)} dx$

(e) $\int_0^{\pi/3} \tan^3 x \sec^5 x dx$

(f) $\int \sqrt{25 - x^2} dx$

2. Find the best possible left, right, midpoint, trapezoidal, and Simpson's approximations to $\int_{-2}^0 f(x) dx$ given the data in the table below.

x	-2	-1.5	-1	-0.5	0
$f(x)$	2	3	6	10	11

3. If you use numerical integration to estimate $\int_a^b \ln x dx$, how would the following be ordered from least to greatest? L_{100} , R_{100} , M_{100} , T_{100} , S_{200} .

What can you say with certainty about where $\int_a^b \ln x dx$ would fit into your ordering?

4. Find bounds for each of the following errors if $I = \int_0^2 e^{-5x} dx$.

(a) $|I - R_{100}|$

(b) $|I - T_{100}|$

(c) $|I - M_{100}|$

5. If $I = \int_0^2 e^{-5x} dx$, how many subdivisions are required to obtain a midpoint sum approximation with error of at most $1/1,000,000$?

6. Use Euler's Method with 3 steps to estimate $y(3/4)$ if $dy/dx = y - 3$ and $y(0) = 1$.

7. Write an integral equal to the area between $y = 2x + 3$ and $y = x^2 + 7x - 3$.

8. Compute the arc length of $y = \sqrt{1 - x^2}$ from $x = 0$ to $x = 1/2$.

9. Consider the region bounded by $y = 0$, $x = 2$, and $y = x^2$. Write an integral equal to the volume of the object created when the region is revolved about

(a) the x -axis

(b) the line $x = 5$

10. Find the solution to $\frac{dy}{dx} = \frac{\cos x}{y^2}$ that passes through $(0, 2)$.

11. The probability density function (pdf) of the weights of newborn toads in a certain pond is given by $f(x) = \frac{k}{(x+1)^4}$, where x is the weight (in ounces). Note that the domain is $x \geq 0$ since no toad can have a negative weight.

(a) What must be the value of k ?

(b) What fraction of the newborn toads weigh more than one ounce?