1a) The four points $(1, 9)$ $(3, -37)$ $(4, 10)$ and $(7, -19)$ are definitely *not* colinear. Find the least-squares line for these points, expressed in the form $y = \beta_0 + \beta_1 x$. Show all your work, from the design-matrix equation through the solution of the least-squares problem.

1b) For your answer in 1a, what are the corresponding four *expected* values corresponding to 1,3, 4 and 7 respectively?

1c) Find the least squares error (the sum-of-squares of the residuals) for the line in (1a)

1d) Find the equation of the line containing $(1, 9)$ and $(7, -19)$. What is the sum of squares of the residuals for this line?

2a) The three points (1,6) (4,3) (5,6) *do* lie on a parabola of the form $y = \beta_0 + \beta_1 x + \beta_2 x^2$. Set up the system of equations you need to solve in order to find β_0, β_1 and β_2, and solve it.

3) Suppose $T : \mathbf{R}^2 \to \mathbf{R}^3$ satisfies $T(\left(\begin{bmatrix}3\\2\end{bmatrix}\right) = \begin{bmatrix}6\\0\\3\end{bmatrix}$ and $T(\left(\begin{bmatrix}5\\1\end{bmatrix}\right) = \begin{bmatrix}4\\1\\2\end{bmatrix}$.

3a) Find $T(\left(\begin{bmatrix}320\\204\end{bmatrix}\right)$.

3b) Find $T(\left(\begin{bmatrix}8\\3\end{bmatrix}\right)$.

4) Let A be the matrix $\begin{bmatrix} 1 & 1 & 2 & -3 \\ 1 & 2 & -3 & 1 \\ 1 & -2 & 17 & -15 \\ 3 & 5 & -4 & -1 \end{bmatrix}$

4a) Find a basis for the column space of A.

4b) Find a basis for $(\text{col}(A))^{\perp}$. Recall that the dot product of any vectors in that space, with any of the column vectors of A, must be 0; this may help you set up the correct matrix equation.

4c) Find a basis for $\text{row}(A)$.

5) Let $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix}$

5a) Do the columns of B form an orthogonal set? Explain.

5b) Find the projection, $\mathbf{b}_{proj}$ of $\mathbf{b} = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 8 \end{bmatrix}$ onto col(B).

5c) Find the vector $\mathbf{z}$ satisfying $\mathbf{b} = \mathbf{b}_{proj} + \mathbf{z}$.

6) Let $M = \begin{bmatrix} 5 & 17 \\ 2 & 7 \end{bmatrix}$

6a) Find the inverse of M using simultaneous row reduction on M and I_2. I want to see the steps; that's the important issue here. Avoid fractions if possible.

6b) What's the (formula for the) inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, in general? And when does it exist?

6c) Does the formula in (6b) give you the same answer as you got in (6a)?

7) Let $A = \begin{bmatrix} 88 & -140 & 110 \\ 90 & -142 & 110 \\ 45 & -70 & 53 \end{bmatrix}$; then A has two eigen values, $\lambda = 3$ and $\lambda = -2$. Diagonalize A; that is, find P and D having the "right properties". You don't have to find P^{-1}.

8) Let $B = \begin{bmatrix} -4 & 7 & 4 \\ 0 & 2 & 0 \\ 2 & 6 & 3 \end{bmatrix}$

8a) Find $\det(B)$. Show your work.

8b) Find the eigenvalues of B. Show your work.

8c) Find $\det(B^{-1})$.

8d) Find $\det(B^3)$.