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I____II____III____IV____V____VI____VII____VIII____IX___ TOTAL ___________

April 12 2005 | Mathematics 206a | Mr. Haines

Multivariable Calculus

Final Examination

I. (10) Let M be parametrized by $f(s,t) = (s+t, s-t, st)$.

A. Calculate a unit normal to M at the point (2, 0, 1).

B. Calculate the tangent plane to M at the point (2, 0, 1).

II. (5) Give an equation of the tangent plane to the graph of $z = xy + x^3 y + x$ at the point where $x = y = 1$.

III. (5) If $f(x, y) = x^3 + xy + y^5$, give the Hessian <u>form</u> for f at $(1, 0)$.

IV. (10) If $\mathbf{g}:\Re^2 \rightarrow \Re^3$ is the parametrization of the surface S in $\Re^3$ with parametrization $\mathbf{g}(s, t) = (s^2, t^2, s+t)$, and $\mathbf{a}$ is the point (4, 4, 4).

A) Compute the Jacobian of $\mathbf{g}$ at the point where $(x, y, z) = \mathbf{a}$.

B) The total derivative of $\mathbf{g}$ at $\mathbf{a}$, $(\mathbf{Dg}(\mathbf{a})) : \Re^2 \rightarrow \Re^3$ is also a surface in $\Re^3$. Give the equation of the tangent plane to the total derivative at the point where $(x, y, z) = \mathbf{a}$.

V. (10) Let R be the triangle with vertices (0, 0), (1, 1), and (1, 2). Evaluate the double integral

$$\iint_R (xy)\,dA$$

VI. (10) Suppose M is the part of the surface of the sphere of radius 5 centered at the origin that is cut off by the plane $z = 4$. M looks like a contact lens or a beanie or a cap.

A. Give a parametrization for ∂M.

B. Give a parametrization for M.

VII. (20) Let **F** be a vector field given by **F**(x, y, z) $= (yze^x, ze^x, ye^x)$.

Let M be the top half of the disk in the xy-plane with center (1, 1) and radius 1. Let C be the boundary of M, oriented counterclockwise. (So C looks like the letter D lying on its back. $C = C_1 + C_2$, where C_1 is a straight line and C_2 is a semi-circle.)

A. Find a potential function for **F**.

B. Evaluate $\int_C \mathbf{F} \bullet d\mathbf{x}$.

C. Evaluate $\int_{C_1} \mathbf{F} \bullet d\mathbf{x}$.

D. Evaluate $\int_{C_2} \mathbf{F} \bullet d\mathbf{x}$.

VIII. (15) The Divergence Theorem (Gauss' Theorem) says $\iiint_S div\mathbf{F}dV = \oiint_{\partial S} \mathbf{F} \bullet \mathbf{n}d\sigma$ Explain the meaning of each of these symbols, i.e. tell what they represent. You don't need to give the conditions they satisfy, although that would be nice.

A. ***F***

B. S

C. ∂S

IX. (15) Let M be the portion of the surface of the sphere with radius $\sqrt{5}$ and center (0, 0, -2) that is above the xy-plane and let **F** be a vector field given by **F**(x, y, z) = (y, -x, e^{xz}) .

A) What is the coordinate equation of this sphere (in terms of x, y, and z)?

B) What is the intersection of this sphere with the xy-plane?

C) Use Stokes's Theorem to calculate $\iint_M curl\,\mathbf{F} \bullet \mathbf{n}\ d\sigma$ by evaluating the line integral of **F** around ∂M , the answer to part B) .