

Math 106, Section D

FINAL EXAM

04/11/07

1. (20 points) Find the arc length. $f(x) = x^2, \quad x \in [-\frac{1}{2}, \frac{1}{2}]$

2. (20 points) Find the volume of the solid of revolution formed when the region bounded by $y = x^2 - 1$ and $y = x + 1$ is revolved about the line $y = -1$.

3. (20 points) A conic gasoline tank is formed by rotating the segment of $y = x, x \in [0, 30]$ about the y -axis. The tank is full, and gasoline weighs 43 lb/ft³. Find the amount of work needed to pump all the gasoline in the tank to a nozzle that is 10 feet above the top of the tank.

4. (20 points) Evaluate the integral. $\int \sin^4 x \cos^2 x dx$

5. (20 points) Evaluate the integral. $\int \frac{dx}{(x^2 - 25)^{3/2}}$

6. (20 points) Evaluate the integral. $\int \frac{dx}{x(x + \sqrt[3]{x})}$

7. (20 points) Determine whether the integral is convergent or divergent. If it converges, evaluate it. If it diverges, give reasons.

$$\int_0^e x^2 \ln x dx$$

8. (20 points) Determine whether the series are convergent or divergent. If they converge, find their sums.

a) $\sum_{n=1}^{\infty} \left(\cos \frac{1}{n} - \cos \frac{1}{n+1} \right)$

b) $\sum_{n=0}^{\infty} \frac{(\ln \pi)^n}{n!}$

c) $\sum_{n=0}^{\infty} \frac{(\cos \pi)^n}{(\ln \pi)^n}$

9. (20 points) Determine whether the series are absolutely convergent, conditionally convergent, or divergent.

a) $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n-1}}$

b) $\sum_{n=0}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$

10. (20 points) Determine the radius and interval of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n3^n}$$

11. (20 points) Find a power series representation for the function.

$$f(x) = \frac{2x}{(1+x)^3}$$

12. (20 points) Determine whether the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, conditionally convergent, or divergent.

a) $a_1 = 2, \quad a_{n+1} = \frac{5n+1}{4n+3} a_n$

b) $a_1 = 1, \quad a_{n+1} = \frac{2+\cos n}{\sqrt{n}} a_n$