

1A. Let $A = \begin{bmatrix} -1 & 4 & -6 \\ 0 & 3 & 0 \\ -2 & 2 & 0 \end{bmatrix}$ and verify that $\begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$ is an eigenvector of A .

What is the corresponding eigenvalue?

$$\begin{bmatrix} -1 & 4 & -6 \\ 0 & 3 & 0 \\ -2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ 0 \end{bmatrix} = 3 \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} \Rightarrow \lambda = 3$$

1B. Fact: Another eigenvalue of A is $\lambda = -4$. Find a basis for the eigenspace of $\lambda = -4$.

$$(A - -4I) = \begin{bmatrix} 3 & 4 & -6 \\ 0 & 7 & 0 \\ -2 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ the solutions of } (A - -4I)\vec{x} = \vec{0}$$

are $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_3 \\ 0 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

where x_3 is free; a basis for the eigenspace of $\lambda = -4$ is $\therefore \left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$

1C. Fact: There are only two distinct eigenvalues for this matrix (so one of them has multiplicity 2). Show that A is diagonalizable; exhibit appropriate P and D that demonstrate this.

Since the eigenspace of $\lambda = -4$ is 1D, it must be that the eigenspace of $\lambda = 3$ is 2D since we're told A is diagonalizable.

$$\text{now, } (A - 3I) = \begin{bmatrix} -4 & 4 & -6 \\ 0 & 0 & 0 \\ -2 & 2 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 3/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{cases} x_1 = x_2 - \frac{3}{2}x_3 \\ x_2 \text{ is free} \\ x_3 \text{ is free} \end{cases}$$

and a basis for the eigenspace for $\lambda = 3$

$$\text{is } \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3/2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\therefore \text{ we have } P = \begin{bmatrix} 1 & -3/2 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \text{ \& } D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

1D. What is the characteristic polynomial of A , in factored form?

$$(\lambda - 3)^2 (\lambda + 4)$$

and can write
 $A = PDP^{-1}$

(Other "arrangements" are possible & the cols. of P are NOT unique)

2A. Fact: The matrix $B = \begin{bmatrix} -22 & 25 & 36 \\ -1 & 4 & 2 \\ -12 & 12 & 20 \end{bmatrix}$ has exactly the same characteristic polynomial as matrix

A in problem 1, yet B is not diagonalizable. What must "go wrong"?

at least one eigenspace must have dimension strictly smaller than the multiplicity of the corresponding eigenvalue. Since the multiplicities of $\lambda=3$ and $\lambda=-4$ are 2 and 1, respectively, this implies the eigenspace for $\lambda=3$ must be (only) 1

2B. Support your conclusion: your answer will be in terms of one of the eigenspaces of B .

Let's find out about $\lambda=3$: $(B-3I) = \begin{bmatrix} -25 & 25 & 36 \\ -1 & 1 & 2 \\ -12 & 12 & 17 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Since there is only one "free variable" in this matrix, the dimension of the eigenspace is indeed only 1, but again the multiplicity of $\lambda=3$ is 2. \therefore P & D such that

$$A = PDP^{-1} \text{ (where } D \text{ is diagonal \& the cds. of } P \text{ are L.I. eigenvectors corresponding to the } \lambda\text{'s on the diagonal of } D\text{) } \underline{\underline{\text{CANNOT BE FOUND}}}$$

(D is "OK." ; P is the problem)

3. Let $C = \begin{bmatrix} 25 & 58 & 11 & 36 \\ 1 & 4 & 1 & 2 \\ 12 & 27 & 5 & 17 \end{bmatrix}$, and let R be the Reduced Row Echelon Form of C .

Find a basis for each of the following: (you may write "same as" if the basis is the same as the answer to an earlier one — eg, "same as 3A"). Various helpful row reductions appear elsewhere on this exam.

3A. $\text{Col}(C)$

R shows the pivot cols of C are

$$\vec{c}_1, \vec{c}_2 \therefore \left\{ \begin{bmatrix} 25 \\ 1 \\ 12 \end{bmatrix}, \begin{bmatrix} 58 \\ 4 \\ 27 \end{bmatrix} \right\}$$

3B. $\text{Col}(C^T)$

$\text{Col}(C^T) = \text{Row}(C)$ so a basis is

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ -1/2 \\ 2/3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1/3 \\ 1/3 \end{bmatrix} \right\}$$

given by the pivot rows of R , that is,

ALTERNATIVELY:
We're given $\text{RREF}(C^T)$ elsewhere; the pivot cols are 1 & 2 saying that

$$\left\{ \begin{bmatrix} 25 \\ 58 \\ 11 \\ 36 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 1 \\ 2 \end{bmatrix} \right\} \text{ is also a basis}$$

note that

$$25 \begin{bmatrix} 1 \\ 0 \\ -1/2 \\ 2/3 \end{bmatrix} + 58 \begin{bmatrix} 0 \\ 1 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 25 \\ 58 \\ 11 \\ 36 \end{bmatrix}$$

confirming that at least one of the 2nd basis elts is a L.C. of the first basis!

3C. $\text{Null}(C)$

from R we see:

$$\left\{ \begin{bmatrix} 1/3 \\ -1/3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2/3 \\ -1/3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

3D. $\text{Null}(C^T)$

from $\text{RREF}(C^T)$, given elsewhere, we see

$$\left\{ \begin{bmatrix} -1/2 \\ 1/2 \\ 1 \end{bmatrix} \right\}$$

3E. $\text{Row}(C)$

note that $\text{Row}(C) = \text{Row}(R) = \text{Col}(C^T)$
so they all have the same bases.

\therefore same as 2B (you may want to write the vectors "sideways")

3F. $\text{Row}(R)$

Same as 2B
and 2E

3G. $\text{Null}(R)$

same as 2C

3H. $(\text{Null}(C))^{\perp}$ The only "non obvious" one!

we need \vec{x} s.t. $\vec{v} \cdot \vec{x} = 0$ where \vec{v} 's are from a basis of $\text{Null}(C)$, that is, solns of

$$\begin{bmatrix} 1/3 & -1/3 & 1 & 0 \\ -2/3 & -1/3 & 0 & 1 \end{bmatrix} \vec{x} = \vec{0}$$

row reduction of THIS yields:

$$\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

3I. $(\text{Col}(C))^{\perp}$ we need \vec{x} 's \perp cols of C , or, equivalently, solns of $C^T \vec{x} = \vec{0}$; that is,

$$\text{Col}(C)^{\perp} = \text{Null}(C^T) \Rightarrow \text{same as 3D}$$

4A. In terms of \vec{x} (the amount produced), C (the consumption matrix) and \vec{d} (the final demand), what is the equation used to model the Leontief Input-Output Model for an economy?

$$\vec{x} = C\vec{x} + \vec{d}$$

4B. Suppose a certain economy has three producing sectors, Math, Music and Food, and an open sector of People who just consume Math, Music and Food while producing nothing. As it turns out, the silly People demand no Math, but lots of Food, 708 units worth, and an amount of Music, S , which we will determine by the end of the problem. Making one unit of Math consumes $4/10$ units of Math, 0.15 of one unit of Music, and an as yet unknown amount f of a unit of Food, whereas Music requires $1/10$ of a unit each of Music and Food, but an unknown amount m of a unit of Math, and Food requires 0.05 , 0.25 and 0.2 units of Math, Music and Food, respectively. The production levels of Math, Music and Food are 360 , 800 , and 1120 units.

Explicitly write out the Leontief Input-Output Model for this system in matrix notation. There will be three unknowns: f , m and S and lots of numbers in the matrix equation you write.

$$\begin{bmatrix} 360 \\ 800 \\ 1120 \end{bmatrix} = \begin{bmatrix} 0.4 & m & 0.05 \\ 0.15 & 0.1 & 0.25 \\ f & 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} 360 \\ 800 \\ 1120 \end{bmatrix} + \begin{bmatrix} 0 \\ S \\ 708 \end{bmatrix}$$

4C. Find f , m and S . *the above matrix equation yields these three linear equations:*

$$360 = 0.4 * 360 + m * 800 + 0.05 * 1120 \Rightarrow m = \frac{360 - 144 - 56}{800} = \frac{160}{800} = 0.2$$

$$800 = 0.15 * 360 + 0.1 * 800 + 0.25 * 1120 + S \Rightarrow S = 800 - 54 - 80 - 280 = 386$$

$$1120 = f * 360 + 0.1 * 800 + 0.2 * 1120 + 708 \Rightarrow f = \frac{1120 - 80 - 224 - 708}{360} = \frac{108}{360} = 0.3$$

4D. Explicitly find the intermediate demands for Math, Music, and Food.

This is given by $C\vec{x}$

$$\text{or, } \begin{bmatrix} .4 & .2 & .05 \\ .15 & .1 & .25 \\ .3 & .1 & .2 \end{bmatrix} \begin{bmatrix} 360 \\ 800 \\ 1120 \end{bmatrix} = \begin{bmatrix} 360 \\ 414 \\ 412 \end{bmatrix} \begin{matrix} \leftarrow \text{Math} \\ \leftarrow \text{Music} \\ \leftarrow \text{Food} \end{matrix}$$

alternatively, $C\vec{x}$ is $\vec{x} - \vec{d}$
 which is $\begin{bmatrix} 360 \\ 800 \\ 1120 \end{bmatrix} - \begin{bmatrix} 0 \\ 386 \\ 708 \end{bmatrix}$
 \leftarrow (same as this)

5. Suppose T is a linear transformation with standard matrix $C = \begin{bmatrix} 25 & 58 & 11 & 36 \\ 1 & 4 & 1 & 2 \\ 12 & 27 & 5 & 17 \end{bmatrix}$ (the same C as in problem 3).

5A. Find any conditions on $\mathbf{b} = (b_1, b_2, b_3)$ (written sideways to save space) such that \mathbf{b} is in the image of T . Explain your answer.

We know $\vec{b} \in \text{Image of } T$

\Leftrightarrow

there is some $\vec{x} \in \text{domain of } T$

s.t. $T(\vec{x}) = \vec{b}$

\Leftrightarrow

$C\vec{x} = \vec{b}$ has a solution.

info elsewhere on this exam tells us

$$[C | I_3] \sim [0000 | 1 -1 -2]$$

which says $0 = b_1 - b_2 - 2b_3$, that is,

\vec{x} exists $\Leftrightarrow b_1 - b_2 - 2b_3$ is 0, otherwise

the underlying system is inconsistent.

- 5B. Give an explicit example of a vector not in the image of T .

an easy example is $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

- 5C. Find $T(\vec{w})$ where (written sideways) $\vec{w} = (2, 0, 0, 1)$.

$$T(\vec{w}) = C\vec{w} = \begin{bmatrix} 25 & & & \\ & \text{etc.} & & \\ & & & 17 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 25 \\ 1 \\ 12 \end{bmatrix} + 1 \begin{bmatrix} 36 \\ 2 \\ 17 \end{bmatrix} = \begin{bmatrix} 50 \\ 2 \\ 24 \end{bmatrix} + \begin{bmatrix} 36 \\ 2 \\ 17 \end{bmatrix} = \begin{bmatrix} 86 \\ 4 \\ 41 \end{bmatrix}$$

5D. Recall the kernel of a linear transformation T is the set of all vectors in the domain which T maps to the zero-vector. Find a basis for this subspace of T .

We know $T(\vec{x}) = \vec{0}$

\Leftrightarrow

$C\vec{x} = \vec{0}$; thus

$\text{Ker}(T) = \text{Null}(C)$. In (3B) of this Exam, we found a basis of $\text{Null}(C)$ to be

$$\left\{ \begin{bmatrix} 1/3 \\ -1/3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2/3 \\ -1/3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

6A. A "catenary" is the name of the curve traced out by a flexible chain suspended at its ends and hanging freely. Galileo declared such a curve is really a parabola, but in fact this is incorrect. Just for fun, I hung one of my wife's necklaces over my computer monitor, which was displaying a coordinate system; a photograph appears below. Sure looks parabolic! The four data points I observed on the curve are at (0.6, 2), (5, -0.5), (6.3, 0) and (8, 2). Determine if they *do* lie on a parabola of the form $y = \beta_0 + \beta_1 x + \beta_2 x^2$ by setting up the appropriate matrix equation and row reducing it. Show all your results to two decimal places. (Hint: one of the four linear equations represented by your matrix equation will be $\beta_0 + \beta_1 \cdot 8 + \beta_2 \cdot 64 = 2$.)

The matrix equation becomes

$$\begin{bmatrix} 1 & 0.6 & 0.36 \\ 1 & 5 & 25 \\ 1 & 6.3 & 36.39 \\ 1 & 8 & 64 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -0.5 \\ 0 \\ 2 \end{bmatrix}$$

$$\underline{X} \vec{\beta} = \vec{y}$$

When put in RREF, the corresponding augmented matrix

$$\text{is } \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \text{ which tells us the system is inconsistent,}$$

ie. these four points do NOT lie on a parabola of the form $y = \beta_0 + \beta_1 x + \beta_2 x^2$.

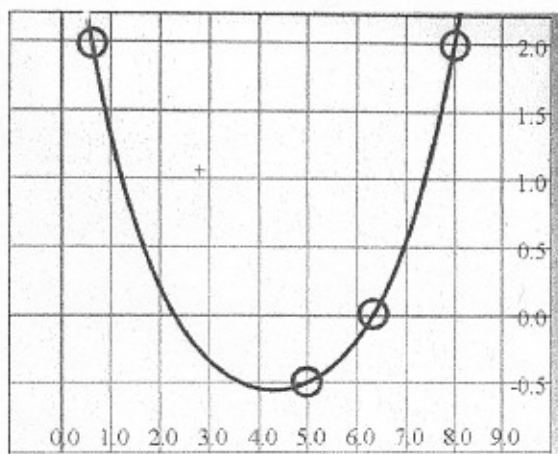


Fig. 1 Necklace Catenary

6B. Find the "best-fit" parabola using the method discussed in class. (Find β_0 , β_1 , and β_2 .)

The method says the best-fit parabola satisfies $\underline{X}^T \underline{X} \vec{\beta} = \underline{X}^T \vec{y}$

you can do the whole problem on your calculator[Ⓢ]; the above line becomes
 (without re-entering any #'s once you enter \underline{X} as matrix A and \vec{y} as matrix B)

$$\begin{bmatrix} 4 & 19.9 & 129.05 \\ 19.9 & 129.05 & 887.26 \\ 129.05 & 887.26 & 6296.43 \end{bmatrix} \vec{\beta} = \begin{bmatrix} 3.50 \\ 14.70 \\ 116.22 \end{bmatrix}; \text{ row reduction yields } \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2.94 \\ 0 & 1 & 0 & -1.67 \\ 0 & 0 & 1 & 0.19 \end{array} \right]$$

(NOTE: SEE NEXT PAGE OF THESE SOLNS so $y = 2.94 - 1.67x + 0.19x^2$ is the best-fit parabola.)
 FOR A GRAPH OF BOTH THE NECKLACE AND

6C. What is the projection of the four \vec{y} -coordinates on the catenary onto the column space of the design matrix in this problem?

That's $\underline{X} \vec{\beta}$; now we know $\vec{\beta}$ is $\begin{bmatrix} 2.94 \\ -1.67 \\ 0.19 \end{bmatrix}$ so multiplying $\underline{X} \vec{\beta}$ gives $\begin{bmatrix} 2.01 \\ -0.66 \\ -0.04 \\ 1.74 \end{bmatrix}$

6D. What are the y coordinates on your best-fit parabola?

this is the same as 6C, ie. these four y -coords

are closer to

$$\begin{bmatrix} 2 \\ -0.5 \\ 0 \\ 2 \end{bmatrix}$$

than any other L.C. of the column of \underline{X} .

6F. What is the "distance" in \mathbb{R}^4 between these two sets of y -coordinates? Question is POORLY WORDED.

if you read "these" as the ones in 6C & 6D the answer is 0.

I meant between $\underline{X} \vec{\beta}$ and \vec{y} ; that distance is $\approx \sqrt{0.01^2 + 1.6^2 + 0.04^2 + 0.26^2} (= .3081)$

7. Let S be a set of vectors in \mathbb{R}^n .

7A. Define what it means for S to be linearly independent.

Let $S = \{\vec{v}_1, \dots, \vec{v}_p\}$, say.

Then S is L.I. \iff the ONLY solution to $x_1\vec{v}_1 + \dots + x_p\vec{v}_p = \vec{0}$ is the trivial solution, $x_1 = \dots = x_p = 0$

7B. Define what it means for S to be orthogonal.

Distinct pairs of members of S are \perp , or, better

yet, $\vec{v}_i \cdot \vec{v}_j = 0$ for $i \neq j$
(or $v_i \perp v_j$ for $i \neq j$)

7C. Suppose S is an orthogonal set of nonzero vectors. Prove S is linearly independent.

Suppose $S = \{\vec{v}_1, \dots, \vec{v}_p\}$ is orthogonal and none of the \vec{v}_i 's is $\vec{0}$.

Suppose that $x_1\vec{v}_1 + \dots + x_p\vec{v}_p = \vec{0}$. Then ("dot" each side with \vec{v}_1 to get...)

$$x_1(\vec{v}_1 \cdot \vec{v}_1) + \dots + x_p(\vec{v}_p \cdot \vec{v}_1) = \vec{0} \cdot \vec{v}_1$$

All dot products $\vec{v}_2 \cdot \vec{v}_1, \vec{v}_3 \cdot \vec{v}_1, \dots, \vec{v}_p \cdot \vec{v}_1$ are 0 because $\vec{v}_2 \perp \vec{v}_1, \dots, \vec{v}_p \perp \vec{v}_1$

So the LHS is simply $x_1(\vec{v}_1 \cdot \vec{v}_1)$. The RHS is just 0 "0 dot anything is 0"

so $x_1(\vec{v}_1 \cdot \vec{v}_1) = 0$. Now, $\vec{v}_1 \neq \vec{0}$ so $(\vec{v}_1 \cdot \vec{v}_1) \neq 0 \therefore x_1 = 0$

Similarly $x_2 = 0, \dots, x_p = 0$. Hence S is L.I.

note well: these are WRONG or at best partially right:

① " $S\vec{x} = \vec{0}$ has only the trivial solution" WRONG b/c S is a SET not a matrix

② " $A\vec{x} = \vec{0}$ has only the trivial soln" WRONG unless you say, "where the columns of A are $\vec{v}_1, \dots, \vec{v}_p$ "

③ "... no free variables" NO this is a result but not the definition.

④ none of the \vec{v}_i 's is a L.I. of the others ... again a RESULT, not the definition.

PROBLEM 6 REVISITED

after obtaining $y = 2.94 - 1.67x + 0.19x^2$

I plotted it behind the necklace.

The result is shown here \rightarrow

Why do you suppose the resulting parabola isn't "symmetrically behind" the necklace?

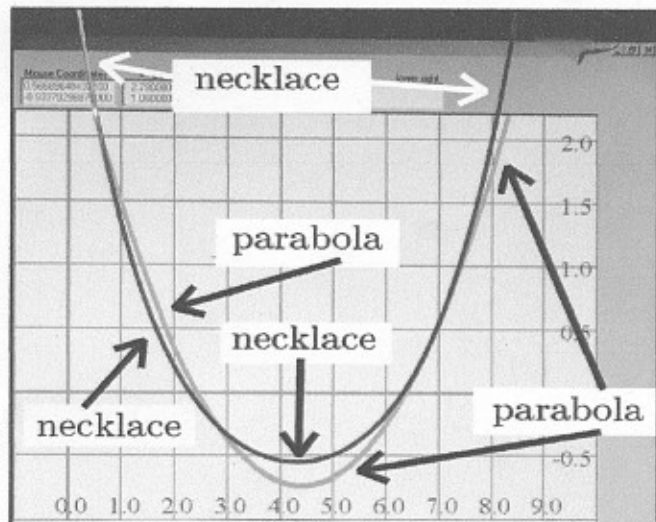


Fig. 2 Necklace Catenary and Best-Fit Parabola

8. What is the dimension of each of the following vector spaces?

8A. P^3

4

8B. $\text{Col}(I_3)$

3

8C. F , our space of continuous functions.

∞

8D. $\{0\}$

0

8E. The eigenspace of $\lambda = -4$ in problem 1.

1

9. Let $M = \begin{bmatrix} 5 & a & 0 \\ 0 & 0 & 2 \\ 0 & 1 & b \end{bmatrix}$.

Find the determinants of each of the following matrices and write your answers in the boxes.

M -10 After one row swap, $M \sim M^* = \begin{bmatrix} 5 & a & 0 \\ 0 & 1 & b \\ 0 & 0 & 2 \end{bmatrix}$ which is upper triangular, so $|M^*| = 5 \cdot 1 \cdot 2 = 10$ and so $\det(M) = -10$

3M -270 each row of M gets multiplied by 3; there are three rows. $\therefore |3M| = 3^3 |M| = 27 \cdot -10 = -270$

M^3 -1000 $|M^3| = \det(M) \cdot \det(M) \cdot \det(M) = (-10)^3 = -1000$

$\begin{bmatrix} 5 & a & 0 \\ 10 & a+a & 2 \\ 10 & 2a+1 & b \end{bmatrix}$ -10

This matrix results from M by adding $2r_1$ to both r_2 and r_3 ; such a change does NOT affect the determinant, so -10 is the answer.

$\begin{bmatrix} 5 & a & a+b \\ 1 & 1 & 1 \\ 6 & a+1 & a+b+1 \end{bmatrix}$ 0

Here $r_1 + r_2 = r_3$, so we can create a row of 0's by $r_3 \leftarrow r_3 - r_2 - r_1$; the determinant is $\therefore 0$

$2M + 3I_3$ 13+78b It's NOT $\det(2M) + \det(3I_3)$ (which would be $-80 + 27$)

you just have to simplify $2M + 3I_3$ and then find the det. Now,

$2M + 3I_3 = \begin{bmatrix} 10 & 2a & 0 \\ 0 & 0 & 4 \\ 0 & 2 & 2b \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 13 & 2a & 0 \\ 0 & 3 & 4 \\ 0 & 2 & 3+2b \end{bmatrix}$ so det is

$13 \begin{vmatrix} 3 & 4 \\ 2 & 3+2b \end{vmatrix} = 13(9+6b-8) = 13(1+6b) = 13+78b$

$M^{-1}M^T$ 1

$\det(M^{-1}M^T) = \det(M^{-1}) \det(M^T) = \frac{1}{\det(M)} \det(M) = 1$ since $\det(M) \neq 0$.

$$\left[\begin{array}{cccc|ccc} 25 & 58 & 11 & 36 & 1 & 0 & 0 \\ 1 & 4 & 1 & 2 & 0 & 1 & 0 \\ 12 & 27 & 5 & 17 & 0 & 0 & 1 \end{array} \right]$$

is row equivalent to

$$\left[\begin{array}{cccc|ccc} 1 & 0 & -1/3 & 2/3 & 0 & -\frac{9}{7} & \frac{4}{21} \\ 0 & 1 & 1/3 & 1/3 & 0 & 4/7 & -1/21 \\ 0 & 0 & 0 & 0 & 1 & -1 & -2 \end{array} \right]$$

$$\left[\begin{array}{ccc|cccc} 25 & 1 & 12 & 1 & 0 & 0 & 0 \\ 58 & 4 & 27 & 0 & 1 & 0 & 0 \\ 11 & 1 & 5 & 0 & 0 & 1 & 0 \\ 36 & 2 & 17 & 0 & 0 & 0 & 1 \end{array} \right]$$

is row equivalent to

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1/2 & 0 & 0 & -1/7 & 1/14 \\ 0 & 1 & -1/2 & 0 & 0 & \frac{18}{7} & -\frac{11}{14} \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -2 & -1 \end{array} \right]$$