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April 10 2006

Mathematics 206a
Multivariable Calculus
Final Examination

Mr. Haines

(5) I. If $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 6 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}$ and θ is the angle between **u** and **v**, what is $\cos\theta$?

(5) II. $\mathbf{A}(t) = (t^2, 1, \frac{1}{t}, t+1)$ is a parametrization of a path in 4-space. Give the equation of the tangent line to this path at $\mathbf{A}(1)$.

(5) III. The points (1, 1, 1), (3, 4, 5), and (2, 2, 4) are in $\Re^3$.

A. Determine whether or not they lie on the same line.

B. If they lie on the same line, give a parametric equation for that line. If they do not lie on the same line, give an equation (either parametric or coordinate) for the plane that contains them.

(12) IV. $S = \{(x,y,z) \mid x^2 + y^2 + z^2 \le 9$ is a solid ball. Note that

$\partial S = \{(x,y,z) \mid x^2 + y^2 + z^2 = 9$ and that the volume of a ball or radius r is $(4/3)\pi r^3$.

A. Given that (2, 1, 2) is on ∂S, give the equation of the tangent plane to ∂S at (2, 1, 2).

B. Calculate **n**, the unit normal vector to ∂S at (2,1,2) .

C. Calculate the surface integral $\oint\!\!\oint_{\partial S} \mathbf{F} \bullet \mathbf{n} \, d\sigma$ for the vector field **F** = (x, y, z) in $\Re^3$.

(10) V. C is the straight line segment in 3-space that starts at (1, 1, 1) and ends at (2, 3, 4).

$\mathbf{F} : \Re^3 \to \Re^3$ has rule $\mathbf{F}(x,y,z) = (z,z,y) - (x,y,x)$.

Compute $\int_C \mathbf{F} \bullet d\mathbf{x}$.

(10) VI. The equations for changing from spherical to rectangular coordinates are:

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta \quad \text{where } \rho^2 = x^2 + y^2 + z^2$$
$$z = \rho \cos\phi$$

Suppose the solid S is the sphere of radius 1 centered at (0, 0, 0) in $\Re^3$. In rectangular coordinates S can be described by
$\{(x, y, z) \mid -1 \le x \le 1; \quad -\sqrt{1-x^2} \le y \le \sqrt{1-x^2}; \quad -\sqrt{1-x^2-y^2} \le z \le \sqrt{1-x^2-y^2}\}$.

In spherical coordinates S can be described by $0 \le \theta \le 2\pi; \quad 0 \le \phi \le \pi; \quad 0 \le \rho \le 1$.

Change variables from rectangular to spherical and then evaluate this triple integral:

$$\iiint_S \sqrt{x^2 + y^2 + z^2}\, dxdydz .$$

(8) VII. The surfaces with equations $z = x^2 + y^2$ and $z = 2x + 2y - 1$ enclose a region of finite volume. Set up but **do not evaluate** a double integral that computes that volume. [Hint: Try completing the square.]

(10) VIII. C is a curve in $\Re^3$ parametrized by $\mathbf{f} : [0,3] \to \Re^3$ with rule $\mathbf{f}(t) = \left(\frac{t^2}{2}, \frac{t^2}{2}, \frac{t^2}{2} \right)$.

Compute the length of C.

(10) IX. Give an example of a conservative vector field, $\mathbf{F}:\Re^3 \to \Re^3$ and prove it is conservative. If C is the curve parametrized by $\mathbf{x}(t) = (3\cos t, 5\sin t, 7)$ for $0 \le t \le 2\pi$,

compute $\int_C \mathbf{F} \bullet d\mathbf{x}$.

(10) X. Given the vector field $\mathbf{F}(x,y,z) = (2xyz, x^2z + 2yz, x^2y + y^2)$, if C is a path in $\Re^3$ parametrized by $\mathbf{c}(t) = \left(\sqrt{t}, \sin \pi t, t^5\right)$ with $0 \le t \le 1$, calculate $\int_C \mathbf{F} \bullet d\mathbf{x}$.

(5) XI. Suppose $f : \Re^3 \to \Re$ with rule $f(x, y, z) = x + y + z^4$. Calculate the directional derivative of f at (1, 1, 1) in the direction parallel to the vector (1, 0, 1).

(5) XII. Suppose $\mathbf{f}(x, y, z) = (x, y, xyz)$ and **a** = (1, 1, 1). Give the total derivative of **f** at **a**.

(5) XIII. For the function $f(x, y, z) = xyz$ at the point (1, 1, 0) compute the first-degree Taylor polynomial.