(5) I. If $u = \begin{bmatrix} 6 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $v = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}$ and $\theta$ is the angle between $u$ and $v$, what is $\cos \theta$?

(5) II. $A(t) = (t^2, 1/t, t + 1)$ is a parametrization of a path in 4-space. Give the equation of the tangent line to this path at $A(1)$. 
(5) III. The points \((1, 1, 1), (3, 4, 5),\) and \((2, 2, 4)\) are in \(\mathbb{R}^3\).

A. Determine whether or not they lie on the same line.

B. If they lie on the same line, give a parametric equation for that line. If they do not lie on the same line, give an equation (either parametric or coordinate) for the plane that contains them.
(12) IV. \( S = \{ (x, y, z) \mid x^2 + y^2 + z^2 \leq 9 \} \) is a solid ball. Note that
\[ \partial S = \{ (x, y, z) \mid x^2 + y^2 + z^2 = 9 \} \] and that the volume of a ball or radius \( r \) is \( \frac{4}{3} \pi r^3 \).

A. Given that (2, 1, 2) is on \( \partial S \), give the equation of the tangent plane to \( \partial S \) at (2, 1, 2).

B. Calculate \( n \), the unit normal vector to \( \partial S \) at (2,1,2).

C. Calculate the surface integral \( \iint_{\partial S} F \cdot n \, d\sigma \) for the vector field \( F = (x, y, z) \) in \( \mathbb{R}^3 \).

(10) V. \( C \) is the straight line segment in 3-space that starts at (1, 1, 1) and ends at (2, 3, 4). \( F : \mathbb{R}^3 \to \mathbb{R}^3 \) has rule \( F(x, y, z) = (z, z, y) - (x, y, x) \).

Compute \( \int_C F \cdot dx \).
VI. The equations for changing from spherical to rectangular coordinates are:

\[ x = \rho \sin \phi \cos \theta \]
\[ y = \rho \sin \phi \sin \theta \quad \text{where} \quad \rho^2 = x^2 + y^2 + z^2 \]
\[ z = \rho \cos \phi \]

Suppose the solid S is the sphere of radius 1 centered at (0, 0, 0) in \(\mathbb{R}^3\). In rectangular coordinates S can be described by

\[ \{(x, y, z) \mid -1 \leq x \leq 1; \quad -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}; \quad -\sqrt{1-x^2-y^2} \leq z \leq \sqrt{1-x^2-y^2}\}. \]

In spherical coordinates S can be described by \(0 \leq \theta \leq 2\pi; \quad 0 \leq \phi \leq \pi; \quad 0 \leq \rho \leq 1\).

Change variables from rectangular to spherical and then evaluate this triple integral:

\[ \iiint_{S} \sqrt{x^2 + y^2 + z^2} \, dx \, dy \, dz. \]
(8) VII. The surfaces with equations $z = x^2 + y^2$ and $z = 2x + 2y - 1$ enclose a region of finite volume. Set up but do not evaluate a double integral that computes that volume. [Hint: Try completing the square.]

(10) VIII. C is a curve in $\mathbb{R}^3$ parametrized by $f : [0, 3] \rightarrow \mathbb{R}^3$ with rule $f(t) = \left( \frac{t^2}{2}, \frac{t^2}{2}, \frac{t^2}{2} \right)$.

Compute the length of C.
(10) IX. Give an example of a conservative vector field, \( \mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3 \) and prove it is conservative. If \( C \) is the curve parametrized by \( \mathbf{x}(t) = (3\cos t, 5\sin t, 7) \) for \( 0 \leq t \leq 2\pi \), compute \( \int_C \mathbf{F} \cdot d\mathbf{x} \).

(10) X. Given the vector field \( \mathbf{F}(x, y, z) = (2xyz, x^2z + 2yz, x^2y + y^2) \), if \( C \) is a path in \( \mathbb{R}^3 \) parametrized by \( \mathbf{c}(t) = (\sqrt{t}, \sin \pi t, t^5) \) with \( 0 \leq t \leq 1 \), calculate \( \int_C \mathbf{F} \cdot d\mathbf{x} \).
(5) XI. Suppose \( f : \mathbb{R}^3 \rightarrow \mathbb{R} \) with rule \( f(x, y, z) = x + y + z^4 \). Calculate the directional derivative of \( f \) at \((1, 1, 1)\) in the direction parallel to the vector \((1, 0, 1)\).

(5) XII. Suppose \( f(x, y, z) = (x, y, xyz) \) and \( a = (1, 1, 1) \). Give the total derivative of \( f \) at \( a \).

(5) XIII. For the function \( f(x, y, z) = xyz \) at the point \((1, 1, 0)\) compute the first-degree Taylor polynomial.