MATH 205A,B LINEAR ALGEBRA - PROF. P. WONG

FINAL EXAM - APRIL 9, 2013

NAME: **Section:**(Circle one) A(1 : 10) B(2 : 40)

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	**Maximum Score**	**Your Score**
1.	21	
2.	23	
3.	22	
4.	21	
5.	21	
6.	21	
7.	21	
Total	150	

1. Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 2 \end{bmatrix} \qquad \text{and} \qquad \vec{b} = \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}.$$

(a)(7 pts) Solve the system $A\vec{x} = \vec{b}$.

(b)(7 pts) Find an orthonormal basis for the column space Col$A$ of A.

(c)(7 pts) Find a matrix Q and an upper triangular matrix R such that $A = QR$ and $Q^T Q = I$.

2. Let

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}.$$

(a)(7 pts) Find the eigenvalues of A and their corresponding eigenspaces.

(b)(7 pts) Is A diagonalizable? Explain.

(c)(9 pts) Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$. Find the $\mathcal{B}$-matrix of the linear transformation T defined by $T(\vec{x}) = A\vec{x}$.

3. Let

$$A = \begin{bmatrix} -1 & 3 & 1 \\ 2 & -5 & -2 \end{bmatrix}.$$

(a)(7 pts) Find a basis for the null space Nul$A$ of A.

(b)(7 pts) Find a basis for the column space Col$A$ of A.

(c)(8 pts) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation such that $T(\vec{x}) = A\vec{x}$. Is T one-to-one? Is T onto? Justify your answer.

4. Suppose that A is a 4×4 matrix with eigenvalues 3 and -1 and that $\dim \mathrm{Col}(A + I) = 1$.

(a)(5 pts) Determine whether A is diagonalizable. Explain. (Hint: what are the dimensions of the corresponding eigenspaces?)

(b)(5 pts) Find $\det(A - 3I)$, the determinant of the matrix $(A - 3I)$. Justify your answer.

(c)(5 pts) Find the characteristic polynomial of A.

(d)(6 pts) Let P be an invertible 4×4 matrix and let $B = PAP^{-1}$. Find the eigenvalues of B. Is B diagonalizable? Justify your answer.

5. Let $T : \mathbb{P}_2 \to \mathbb{R}$ be given by

$$T(a + bt + ct^2) = b$$

where a, b, c are real numbers.

(a)(7 pts) Show that T is a linear transformation.

(b)(7 pts) Find Ker$T$, the kernel of T.

(c)(7 pts) What is the dimension of Ker$T$? Justify your answer.

6. Let $\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \vec{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \vec{y} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $W = \text{Span}\{\vec{u}, \vec{v}\}$.

(a)(7 pts) Find an orthogonal basis for W.

(b)(7 pts) Find the closest vector in W to $\vec{y}$.

(c)(7 pts) Find a vector in $W^{\perp}$. What is the shortest distance between $\vec{y}$ and W?

7. Let

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 1 \end{bmatrix}.$$

Two of the three eigenvalues of A are 5 and -1.

(a)(7 pts) Show that $\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$ is an eigenvector of A. Find the corresponding eigenvalue.

(b)(7 pts) Find the eigenspaces corresponding to $\lambda = 5$ and $\lambda = -1$.

(c)(7 pts) Find an orthogonal matrix P that orthogonally diagonalizes A.