

MATH106A,B CALCULUS II - PROF. P. WONG

FINAL EXAM - APRIL 9, 2008

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	Maximum Score	Your Score
1.	18	
2.	18	
3.	14	
4.	17	
5.	21	
6.	16	
7.	21	
Total	125	

1. Evaluate each of the following integrals.

(9 pts.)(a)

$$\int_1^e \frac{(\ln z)^2}{z} dz$$

(9 pts.)(b)

$$\int \frac{x+2}{x^2-2x-3} dx.$$

2. (11 pts.)(a) Let $k > 0$ be a positive constant. Evaluate the improper integral

$$\int_0^{\infty} x e^{-kx} dx$$

(7 pts.)(b) Consider the function

$$f(x) = \begin{cases} x e^{-kx}, & \text{if } x \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

Based upon (a), what value of k (k positive) is $f(x)$ a probability density function? Justify your answer.

3. Consider the following Initial Value Problem.

$$y' = x(1 + y^2), \quad y(1) = 1.$$

(8 pts.)(a) Use the method of separation of variables to solve this IVP.

(6 pts.)(b) Estimate the value $y(2)$ (when $x = 2$) of the solution using Euler's method with two steps with initial point $(1, 1)$. DO THIS BY HAND.

(17 pts.)4. Find the volume of the solid formed when the region bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 5$ is revolved around the line $x = 5$.
[First sketch a picture of the region.]

5. Determine whether each of the following series converges or diverges.

Justify your answer.

(7 pts.)(a)

$$2 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$$

(7 pts.)(b)

$$\sum_{k=1}^{\infty} \frac{k}{3k+1}$$

(7 pts.)(c)

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$$

6. For each of the following series, determine whether the series converges absolutely, conditionally, or neither.

(8 pts.)(a)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{e^n}$$

(8 pts.)(b)

$$\sum_{n=3}^{\infty} \frac{(-1)^{n+1} n}{n^2 - 1}$$

7.

(7 pts.)(a) Let $f(x) = \frac{2x}{(1+x^2)^2}$. Use the Maclaurin series for $\frac{1}{1-t}$ to find the Maclaurin series for $f(x)$. [Hint: Is $f(x)$ the derivative of some function?]

(5 pts.)(b) Find the fourth order Maclaurin polynomial for $f(x) = \frac{2x}{(1+x^2)^2}$.

(9 pts.)(c) Find the interval of convergence for the following power series.
[Don't forget to check the endpoints.]

$$\sum_{n=1}^{\infty} \frac{(3x+1)^n}{n2^n}$$