NAME:

Math 106C&D - Professor Shor
Final Exam, April 9, 2008

Instructions: Show all of your work and **circle your final answers**. Your work should flow logically and be easy to follow. Cross out any work that you do not want considered. Calculators are allowed, but notes and books are not.

Problem	Max	Score
1	12	
2	12	
3	8	
4	12	
5	10	

Problem	Max	Score
6	8	
7	16	
8	10	
9	12	
TOTAL	100	

LETTER GRADE:

1. (12 points) Solve the IVP $y' = \dfrac{x \sin x}{3y^2}$, $y(\pi) = 0$.

2. (12 points) Evaluate $\displaystyle\int \frac{dx}{4 - x^2}$.

3. (8 points) Write down an integral that gives the length of the curve $y = x^2$ from $x = 0$ to $x = 2$. (Do not evaluate the integral.)

4. (12 points) Consider the region in the xy-plane that is bounded by the graphs of $y = x^2$, $y = 0$, and $x = 2$. Suppose this region is rotated around the y-axis. Write down an integral that is equal to the volume of the resulting solid. (Do not evaluate the integral.)

5. (10 points) Find the radius and interval of convergence of $\sum_{k=0}^{\infty} \frac{(x-1)^k}{(k+1)4^k}$.

6. (8 points) Evaluate the following series or explain why it diverges: $\sum_{k=1}^{\infty} \frac{1}{3^{k+1}}$. State the name of the convergence test that you use. (If you determine that this series converges, give the exact value that it converges to.)

7. (8 points each) Determine whether the following series converge or diverge, and state the name of the convergence test that you use.

   (a) $\sum_{k=0}^{\infty} \frac{k+3}{2k+1}$.

   (b) $\sum_{k=1}^{\infty} \frac{\arctan k}{1+k^2}$.

8. (10 points) Find the power series centered at $x_0 = 0$ for $f(x) = \ln(1+x)$. You do not need to calculate the radius or interval of convergence.

   (Hint: There are two ways to do this - one by calculating the Taylor series for $f(x)$, and the other by starting with the power series of $\frac{1}{1-x}$ and manipulating it.)

9. (12 points) Let $g(x) = x^2 \sin(2x)$.

   (a) Write out the first 4 terms of the power series of $g(x)$.

   (b) Evaluate $g^{(7)}(0)$.

   (c) Evaluate $g^{(8)}(0)$.

Potentially useful formulæ

- $|I - L_n| \leq \frac{K_1(b-a)^2}{2n}$.
- $|I - R_n| \leq \frac{K_1(b-a)^2}{2n}$.
- $|I - T_n| \leq \frac{K_2(b-a)^3}{12n^2}$.
- $|I - M_n| \leq \frac{K_2(b-a)^3}{24n^2}$.

- $\int_a^b \sqrt{1 + (f'(x))^2}\, dx$

- $\sin^2 x + \cos^2 x = 1$.
- $\tan^2 x + 1 = \sec^2 x$.
- $\frac{d}{dx}(\tan x) = \sec^2 x$.
- $\frac{d}{dx}(\sec x) = \sec x \tan x$.

- $\int \sin^n(x)\, dx = \frac{-\sin^{n-1}(x)\cos(x)}{n} + \frac{n-1}{n}\int \sin^{n-2}(x)\, dx,$ for $n > 0$.
- $\int \cos^n(x)\, dx = \frac{\cos^{n-1}(x)\sin(x)}{n} + \frac{n-1}{n}\int \cos^{n-2}(x)\, dx,$ for $n > 0$.
- $\int \tan^n(x)\, dx = \frac{\tan^{n-1}(x)}{n-1} - \int \tan^{n-2}(x)\, dx,$ for $n \neq 1$.
- $\int \sec^n(x)\, dx = \frac{\sec^{n-2}(x)\tan(x)}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}(x)\, dx,$ for $n \neq 1$.
- $\int \sec(x)\, dx = \ln|\sec(x) + \tan(x)| + C$.

- $P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$.
- $|f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n+1)!}|x - x_0|^{n+1}$.

- $\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$ on $(-\infty, \infty)$.
- $\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$ on $(-\infty, \infty)$.
- $\arctan x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots$ on $[-1, 1]$.