1. Evaluate the following.

(a) \[ \int 7x^2 \cos(x^3) \, dx = \frac{7}{3} \int \cos \omega \, d\omega = \frac{7}{3} \sin \omega + C \]

(b) \[ \int 7x^2 \cos(x^3) \, dx = \frac{7}{3} \int \omega \cos \omega \, d\omega = \frac{7}{3} \left[ \omega \sin \omega - \frac{1}{3} \sin \omega \, d\omega \right] = \frac{7}{3} \left( x^3 \sin(x^3) + \cos(x^3) \right) + C \]

(c) \[ \int_0^\infty 3xe^{-3x} \, dx = \lim_{b \to \infty} \left[ \frac{3x \cdot e^{-3x}}{-3} - \frac{3 \cdot e^{-3x}}{-3} \right] \]

\[ = \lim_{b \to \infty} \left( \frac{-b^2}{e^{3b}} - \frac{1}{3e^3} \right) - \left( 0 - \frac{1}{3e^0} \right) = \frac{1}{3} \]

(d) \[ \int_0^b \frac{1}{(x-4)^2} \, dx = \lim_{b \to 4^+} \left[ \int_0^b (x-4)^{-2} \, dx \right] = \lim_{b \to 4^+} -\frac{1}{x-4} \bigg|_0^b \]

\[ = \lim_{b \to 4^+} -\frac{1}{x-4} \bigg|_0^b = \lim_{b \to 4^-} -\frac{1}{b-4} - \frac{1}{0-4} = \infty \Rightarrow \text{integral diverges} \]

(e) \[ \int \frac{2x+5}{x^2+4x+3} \, dx \]

Partial Fractions:

\[ \frac{2x+5}{x^2+4x+3} = A \cdot \frac{1}{x+1} + B \cdot \frac{1}{x+3} \]

2x+5 = A(x+3) + B(x+1)

Let x = -1 \Rightarrow 3 = A \cdot 2 + B \cdot 0 \Rightarrow A = \frac{3}{2}

Let x = -3 \Rightarrow -1 = A \cdot 0 + B(-2) \Rightarrow B = \frac{1}{2}

\[ \text{So, our integral is:} \int \left( \frac{3/2}{x+1} + \frac{1/2}{x+3} \right) \, dx = \frac{3}{2} \ln|x+1| + \frac{1}{2} \ln|x+3| + C \]

Or, use Table #27.
2. Use a comparison to show whether each of the following converges or diverges.

(a) \( \int_1^\infty \frac{7 + 5 \sin x}{x^2} \, dx \) is like \( \int_1^\infty \frac{\text{constant}}{x^2} \), so should converge (p=2).

\[
\frac{7 + 5 \sin x}{x^2} \leq \frac{7 + 5 \cdot 1}{x^2} = 12 \cdot \frac{1}{x^2}, \text{ so } \int_1^\infty \frac{7 + 5 \sin x}{x^2} \, dx \leq 12 \int_1^\infty \frac{1}{x^2} \, dx, \text{ which is known to converge. So, our } \int \text{ must converge too.}
\]

(b) \( \int_1^\infty \frac{1 + 3x^2 + 2x^3}{\sqrt{10x^6 + 17x^6}} \, dx \) is like \( \int_1^\infty \frac{x^3}{3x^4} \, dx = \int_1^\infty \frac{x^3}{x^4} \, dx = \int_1^\infty \frac{1}{x} \, dx \), so should diverge (p=1).

\[
\frac{1 + 3x^2 + 2x^3}{\sqrt{10x^6 + 17x^6}} = \frac{2x^3}{3} \cdot \frac{3}{10x^6 + 17x^6} > \frac{2x^3}{3} \cdot \frac{3}{10x^6 + 17x^6} \quad \text{make numerator smaller and denominator larger.}
\]

3. If you use numerical integration to estimate \( \int_a^b \ln x \, dx \), how would the following be ordered from least to greatest? LEFT(100), RIGHT(100), MID(100), TRAP(100), SIMP(100)

- \( L \)
- \( T \)
- \( S \)
- \( M \)
- \( R \)

What can you say with certainty about where \( \int_a^b \ln x \, dx \) would fit into your ordering?

- Somewhere between \( T \) and \( M \)
- We can't say whether Simpson's \( S \)

4. Find the best possible left, right, midpoint, trapezoidal, and Simpson's approximations to \( \int_{-2}^0 f(x) \, dx \) given the data in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1.5</th>
<th>-1</th>
<th>-0.5</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

- \( \Delta x = \frac{1}{2} \) for \( L \), \( R \), \( T \)
- \( \Delta x = 1 \) for \( M \), \( S \)

\[
L(2) = \frac{1}{2}[f(-2) + f(-1)] = 10.5
\]

\[
P(2) = \frac{1}{2}[f(-1) + f(-0.5)] = 15
\]

\[
T(2) = \frac{1}{2}[(L(2) + R(2))] = 12.75
\]

\[
M(2) = \frac{1}{2}[(L(2) + R(2))] = 13
\]

\[
S(2) = \frac{2M(2) + T(2)}{3} = \frac{2 \cdot 13 + 12.5}{3} = 12.83
\]
5. Consider the region bounded by \( y = 0, x = 0, x = 2, \) and \( y = x^2 \). Write an integral equal to the volume of the shape created when the region is rotated about

(a) the \( x \)-axis

\[
\text{Vol of slice} \approx \pi r^2 \Delta x = \pi y^2 \Delta x = \pi x^4 \Delta x
\]

\[
\text{Total Vol} = \pi \int_0^2 x^4 \, dx
\]

(b) the line \( x = 5 \)

\[
\text{Vol of slice} \approx \pi (\text{outer rad})^2 \Delta y - \pi (\text{inner rad})^2 \Delta y
\]

\[
\approx \pi (5 - x)^2 \Delta y - \pi (3)^2 \Delta y
\]

\[
\approx \pi (5 - 5)^2 \Delta y - \pi (5 - 3)^2 \Delta y
\]

\[
\text{Total Vol} = \pi \int_0^4 [(5 - 5)^2 - (5 - 3)^2] \, dy
\]

6. The probability density function \( p(x) = 3e^{-3x} \) gives the expected wait time (in minutes) at a fast food restaurant.

(a) What is the probability that you will have to wait less than 2 minutes?

\[
\int_0^2 3e^{-3x} \, dx = \left. -e^{-3x} \right|_0^2 = -e^{-6} + 1 \approx \boxed{.9975}
\]

(b) What is the median wait time?

\[
\text{Find } P(x) = \int_0^x 3e^{-3x} \, dx = \frac{3e^{-3x}}{-3} \bigg|_0^x = -e^{-3x} + 1 \text{ and then solve } P(x) = 1/2.
\]

\[
S_1 = \frac{1}{2} = -e^{-3x} + 1 \Rightarrow e^{-3x} = \frac{1}{2} \Rightarrow -3x = \ln \left( \frac{1}{2} \right) \Rightarrow x = \frac{\ln \frac{1}{2}}{-3} \approx 0.2310
\]

(c) What is the mean (average) wait time?

\[
\text{Mean} = \int_0^\infty x \cdot p(x) \, dx = \int_0^\infty 3xe^{-3x} \, dx = \boxed{\frac{1}{3}} \text{ from } \#1(c).
\]

7. The beach on your waterfront property is eroding. Each year the amount that washes away is 90% of the previous year's. Last year, 4 feet of the beach washed away. So, this year \( 4(.9) \)

How much will wash away in the next ten years (this year and nine more)? \( 3.6 \) ft will wash away.

\[
3.6 + 3.6(.9) + 3.6(.9)^2 + \cdots + 3.6(.9)^9
\]

\[
= \frac{a(1 - r^n)}{1 - r} = \frac{3.6(1 - .9^{10})}{1 - .9} \approx \boxed{23.45 \text{ ft}}
\]

What is the closest to the water that you can now safely build a new house if you plan to live there a long time?

\[
3.6 + 3.6(.9) + 3.6(.9)^2 + \cdots = \frac{a}{1 - r} = \frac{3.6}{1 - .9} = \boxed{36 \text{ feet}}
\]

Since \( |r| < 1 \)
8. Decide if each of the following converges or diverges. Give an explanation for your answers.

a) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}} = \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots \]
   Terms alternate in sign, terms decrease in size, and terms \( \rightarrow 0 \), so Alternating Series Test says series \( \text{converges} \).

b) \[ \sum_{n=1}^{\infty} \frac{(2n)!}{3^n(n!)^2} \]
   Ratio Test
   \[ L = \lim_{n \to \infty} \left| \frac{(2(n+1))!}{3^{n+1}((n+1)!)^2} \cdot \frac{3^n((n!)^2}{(2n)!} \right| = \lim_{n \to \infty} \frac{4n^2 + 6n + 2}{3n^2 + 6n + 3} = \frac{4}{3} > 1 \Rightarrow \text{series diverges} \]
   by L'Hopital's Rule or by looking at dominant terms or by dividing all terms by \( n^2 \).

(c) \[ \sum_{n=1}^{\infty} \frac{1}{100 + \frac{1}{n^5}} \]
   terms \( \rightarrow \frac{1}{100} \neq 0 \), so series \( \text{diverges} \).
   by \( n \text{th} \) term test.
   (You're adding up infinitely many \( \frac{1}{100} \) s.)

(d) \[ \sum_{n=1}^{\infty} \frac{5n^8 + 4n^6}{11n^6 + 13n^2} \]
   is like \[ \sum_{n=1}^{\infty} \frac{\frac{n^4}{11n^6}}{\frac{1}{n^2}} = \sum_{n=1}^{\infty} \frac{1}{n^2} \]
   which is known to converge, so our \( \sum \) must \( \text{converge too} \).

Comparison Test:
original \( \sum \) \( \leq \sum_{n=1}^{\infty} \frac{\sqrt{5n^8 + 4n^6}}{11n^6} = \sum_{n=1}^{\infty} \frac{3n^4}{11n^6} = \sum_{n=1}^{\infty} \frac{3}{11n^2} \]
   is known to converge, so our \( \sum \) must \( \text{converge too} \).

(e) \[ \sum_{n=2}^{\infty} \frac{1}{n(ln(n))^2} \]
   Integral Test:
   \[ \int_{2}^{\infty} \frac{1}{x(ln(x))^2} \, dx = \left[ -\frac{1}{ln(x)} \right]_{2}^{\infty} = 1 \]
   Since \( \int \) converges, the \( \sum \) must \( \text{converge too} \).

See old exams and quizzes at http://abacus.bates.edu/~etowne/mathresources.html