Rabbits were brought to a remote deserted island some years ago. The growth of the rabbit population is governed by the following logistic differential equation.

\[
\frac{dR}{dt} = 0.1R(1000 - R)
\]

Here, \(R = R(t)\) denotes the population of rabbits at time \(t\).

(i) What will the rabbit population be in the long run? (What is the equilibrium solution?)

We know that the logistic equation has a non-trivial (stable) equilibrium when \(R \neq 0\) and \(\frac{dR}{dt} = 0\). It follows that \(R = 1000\) is the stable solution. Thus, in the long run, \(R(t) \rightarrow 1000\).

Now, suppose bobcats were later brought to this island and a population model for the rabbits and for the bobcats is given by the following system.

\[
\frac{dR}{dt} = R \left(2 - R - \frac{B}{3}\right)
\]

\[
\frac{dB}{dt} = B \left(1 - R - \frac{B}{2}\right)
\]

where \(B\) and \(R\) denote the population of the bobcats and of the rabbits (at time \(t\)) respectively.

(ii) Give the equations of the nullclines.

The nullclines are precisely the lines in the \(B - R\) plane where the slope fields are horizontal or vertical. By setting \(\frac{dR}{dt} = 0\), we obtain \(R = 0\) and \(B = 3(2 - R)\). By setting \(\frac{dB}{dt} = 0\), we obtain \(B = 0\) and \(B = 2(1 - R)\). Thus, the nullclines are given by \(R = 0, B = 0, B = 3(2 - R)\) and \(B = 2(1 - R)\).
(iii) For what values of $B$ and $R$ will they co-exist happily ever after without any change in size in their respective populations? (Zero values are allowed.)

To find all equilibrium points, we look for pairs $(B, R)$ at which both $\frac{dB}{dt}$ and $\frac{dR}{dt}$ are zero. The two lines $B = 3(2 - R)$ and $B = 2(1 - R)$ only intersect outside of the first quadrant and since $B \geq 0$ and $R \geq 0$, this intersection is not a feasible equilibrium. The only equilibrium points other than $(0, 0)$ are $(0, 2)$ and $(2, 0)$. 