Math 105: Review for Final Exam, Part II

1. Consider the function $f(x) = x^6 - 2x^3$ on the interval $[-2, 2]$.
   (a) Find the $x$- and $y$-coordinates of any and all local extrema and classify each as a local maximum or local minimum.

   (b) Find the $x$- and $y$-coordinates of any and all global extrema and classify each as a global maximum or global minimum.

   (c) Find the $x$-coordinate(s) of any and all inflection points.

2. How would your answers to the previous question have changed if the domain of $f$ were all reals?

3. Use Newton’s Method to find a root of $f(x) = x^3 - x + 1$ correct to three decimal places.
4. Your company is mass-producing a cylindrical container. The flat portion (top and bottom) costs 3 cents per square inch and the curved (lateral) portion costs 5 cents per square inch. If your budget is $9.00 per container, what dimensions will give the largest volume?

area of circle = \( \pi r^2 \)

lateral area of cylinder = \( 2\pi rh \)

volume of cylinder = \( \pi r^2 h \)

5. You are watching a plane flying toward your position at a constant height of 3 miles and a speed of 500 miles per hour relative to the ground. At the moment when the plane is 5 miles from you (diagonally), at what rate is the angle of your vision toward the plane changing?
6. State the Intermediate Value Theorem and use it to show that \( f(x) = 4x^7 + 3x^3 - 5 \) has a root in \([0, 1]\).

7. What (if anything) does the Extreme Value Theorem say about \( f(x) = x^2 \) on each of the following intervals?
   (a) \([1, 4]\)
   (b) \((1, 4)\)

8. State the Mean Value Theorem and find the value of the constant \( c \) that the theorem specifies for 
   \( f(x) = x^3 + x \) on \([0, 3]\).

9. Find the following.
   (a) \( \int_1^7 \frac{3}{x} \, dx \)
   (b) \( \int_0^2 e^{3x} \, dx \)
   (c) \( \int_{-2}^2 \sqrt{4 - x^2} \, dx \)
   (d) \( \frac{d}{dx} \int_1^x \sin \sqrt{t} \, dt \)

10. Water is leaking out of a tank at a decreasing rate. Find an overestimate and underestimate for the 
    total amount that leaked out during these 8 minutes.

    | time (min) | 0  | 2  | 4  | 6  | 8  |
    |------------|----|----|----|----|----|
    | rate (gal/min) | 15 | 11 | 8  | 4  | 3  |
11. The rate of change of a room’s temperature is \( r(t) = t^2 - 9 \) degrees per hour on the interval \([0, 4]\) hours. At \( t = 0 \), the temperature is 70 degrees. (Remember that \( r \) is the derivative of the temperature function.)

(a) When on this interval is the temperature rising? falling?

(b) What is the maximum temperature on this interval and when does it occur?

(c) What is the minimum temperature on this interval and when does it occur?

(d) What is the average rate of change of the temperature on this interval?

12. Use sigma notation to express \( L_{10} \) as an approximation to \( \int_{20}^{60} \ln x \, dx \).