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April 2 2004

Mathematics 206a
Multivariable Calculus
Examination #3

Mr. Haines

(10) I. Calculate the value of $\int_0^1 \int_0^x \int_0^{x+y} z \, dzdydx$.

(10) II. Give a function $\mathbf{f} : R \subset \Re^2 \to \Re^3$ that is a parametrization of the piece of the surface with equation x - xy + z = 10 that lies above the region in the xy-plane bounded by x = 0, x = 1, y = 0, and y = x^2 .

(20) III. If C is the closed curve formed by the unit circle oriented counterclockwise, with parametrization $f(t) = (\cos t, \sin t)$ for $0 \le t \le 2\pi$.

A. What is the value of $\oint_C u(x, y)dL$ if $u(x, y) = \dfrac{1}{x^2 + y^2}$?

B. What is the value of $\int_C \mathbf{F} \bullet d\mathbf{x}$. if $\mathbf{F}(x, y) = \left(\dfrac{x}{x^2 + y^2}, \dfrac{y}{x^2 + y^2} \right)$?

(20) IV. Suppose **F** is the vector field given by **F** (x, y, z) = x**i** . If C is the straight line segment from (1, 1, 1) to (3, 4, 5),

A. Give a parametrization for C.

B. Calculate $\int_C \mathbf{F} \bullet d\mathbf{x}$.

(10)V. $\mathbf{f} : R \subset \Re^2 \to \Re^3$ defined by **f**(s, t) = (s , cos t, sin t) with $0 \le s \le 4$ and $0 \le t \le 2\pi$ is a parametrization of a surface M in $\Re^3$.

Calculate the surface integral $\iint_M (x + z) d\sigma$.

(10) VI. Compute the line integral of the vector field **F** over the curve C, where **F**(x, y) = (y, x^2) and C is the boundary of the right triangle with vertices (0, 0), (0, 2), and (1, 0) oriented counter-clockwise.

(20) VII. Given the vector field **F** (x, y, z) = $(y, x, 0)$,

A) Prove that **F** is path independent $\Re^3$ by finding a potential function for **F** .

B) If C is a path in $\Re^3$ parametrized by **c** (t) = $\left(\frac{t^4}{4}, \sin^3\left(\frac{t\pi}{2}\right), 0\right)$ with

$0 \le t \le 1$ calculate the line integral $\int_C \mathbf{F} \bullet d\mathbf{x}$.