NAME:
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Water leaks from a vertical cylindrical tank through a small hole in its base at a rate proportional to the square root of the volume of water remaining. The tank initially contains 200 liters and 20 liters leak out during the first day. Let $V(t)$ be the volume of water in the tank at time $t$.

From the hypothesis, the differential equation is $\frac{dV}{dt} = k\sqrt{V}$ for some (negative) constant $k$.

(a) When will the tank be half empty?

By separating the variables, we have $\int V^{-1/2}dV = \int k\ dt$. It follows that $2V^{1/2} = kt + C$ or $V = (At + B)^2$ for some constants $A$ and $B$. Since $V(0) = 200$, $B = \sqrt{200}$. After 1 day, $V(1) = 180$ so $(A + \sqrt{200})^2 = 180$. It follows that $A = \sqrt{180} - \sqrt{200}$. Now, when $V(t) = 100$ (half empty), $t = \frac{10 - \sqrt{200}}{\sqrt{180} - \sqrt{200}} \approx 5.708$ days.

(b) How much water will there be after 4 days?

After 4 days, $V(4) = ((\sqrt{180} - \sqrt{200}) \cdot 4 + \sqrt{200})^2 \approx 126.32$ liters.