Math 205A Quiz 09 page 1 March 28, 2008 NAME suggested sol'ns

1. Suppose an economy has three producing sectors: agriculture, meats, and processed foods. The open sector consists of people who just consume ("eat") all these foods. The four sectors are thus A, M, P and E, respectively. Suppose to produce one unit of output, A requires 0.22 units of its own output, and 0.13 units of M and 0.1 units of P. Making one unit of M requires 0.19, 0.11, and 0.08 units of A, M, and P, resp., and a unit of P consumes 0.1, 0.07 and 0.3 units of A, M, and P, resp. The final demand by the E sector is 40, 30, and 80 units of A, M, and P, resp.

1A. Find the consumption matrix C.

$$\begin{array}{c c} & \begin{array}{ccc} A & M & P \end{array} \\ \begin{array}{c} A \\ M \\ P \end{array} & \begin{bmatrix} 0.22 & 0.19 & 0.1 \\ 0.13 & 0.11 & 0.07 \\ 0.10 & 0.08 & 0.3 \end{bmatrix} \end{array}$$

1B. Find $I - C$ and write it here; store it as $[B]$ in your calculator.

(note: a "during the quiz" request was to set calculators to show THREE decimal places)

$$\begin{bmatrix} .780 & -.190 & -.100 \\ -0.130 & .890 & -.070 \\ -0.100 & -0.080 & .700 \end{bmatrix}$$

note that all main-diag. elts are NON-negative; all others are NON-POSITIVE

1C. Use your calculator to find $(I - C)^{-1}$ and write it here. (Just use $[B]$ and the "x^{-1}" key)

$$(I-C)^{-1} = \begin{bmatrix} 1.363 & 0.311 & 0.226 \\ 0.216 & 1.183 & 0.149 \\ 0.219 & 0.180 & 1.478 \end{bmatrix}$$

1D. Let $\mathbf{d}$ be the final demand vector. In terms of C and $\mathbf{d}$, what is the equation which we set up to find the production vector $\mathbf{x}$?

$$\boxed{\vec{x} = C\vec{x} + \vec{d}}$$

is the set up from the model.

($\vec{x} = (I-C)^{-1}\vec{d}$ is the SOLUTION)

1E. Find the production vector $\mathbf{x}$. (Hint: put the final demand vector $\mathbf{d}$ into your calculator as a matrix (say $[D]$) and do an appropriate matrix multiplication.

that multiplication is $(I-C)^{-1}\vec{d} = [B]^{-1}[D] = \begin{bmatrix} 81.918 \\ 56.087 \\ 132.398 \end{bmatrix}$

1F. *BONUS!* It turns out that $(I_3 + C + C^2 + C^3 + C^4 + C^5)\,\mathbf{d}$ is $\begin{bmatrix} 81.154 \\ 55.618 \\ 131.592 \end{bmatrix}$. What is the connection between this fact and your work in (1A-1E)?

We mentioned in class that (given the right conditions on C)

$$\lim_{n\to\infty} \left(I_3 + C + C^2 + C^3 + \cdots + C^n \right) = (I-C)^{-1},$$

so $(I_3 + C + C^2 + \cdots + C^5)\vec{d}$ should be an approximation to $(I-C)^{-1}\vec{d}$

and indeed the vectors in 1E & 1F are close.

quiz continues other side

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2. Let $A = \begin{bmatrix} 5 & 6 & -6 \\ 3 & 8 & -6 \\ 3 & 6 & -4 \end{bmatrix}$.

2A. It's a fact that $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of A. What is the corresponding eigenvalue? (An easy calculation)

multiply A by $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ to find out:

$$A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}, \text{ so } \lambda = 5$$

2B. It's also true that $\lambda = 2$ is an eigenvalue of A. If possible, find matrices P and D that show A is diagonalizable, or explain why A is not diagonalizable. Show all your work.

Let's find a basis for the eigenspace of 2. (If the dimension is 2, great. Otherwise either 1) dim(eigspace(5)) = 2 OR 2) there's another eigenvector OR 3) A won't be diagonalizable.)

now, $(A-2I) =$

$$\begin{bmatrix} 3 & 6 & -6 \\ 3 & 6 & -6 \\ 3 & 6 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and a basis for the nullspace of THIS matrix is $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$,

(which is a basis for the eigenspace of $\lambda = 2$)

so A is diagonalizable and $A = PDP^{-1}$

where $P = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ & $D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (for example)