Math 205B&C 03/27/09 Quiz 07 page 1 **Name** Suggested Soln's **8am** **1:10 pm**

1. Suppose that $\mathbf{v}_1 = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} 1 \\ x \\ y \end{bmatrix}$; let $\mathbf{s} = \begin{bmatrix} -11 \\ 18 \\ 51 \end{bmatrix}$.

1a. Explain why $\mathbf{v}_1 \perp \mathbf{v}_2$. $\vec{v}_1 \cdot \vec{v}_2 = (3\cdot 3) + (-4\cdot 1) + (5\cdot -1) = 9-4-5 = 0$

1b. Find a unit vector in the direction of $\mathbf{v}_2$. $\|\vec{v}_2\| = \sqrt{3^2+1^2+1^2} = \sqrt{11}$ $\therefore$ $\frac{1}{\sqrt{11}}\vec{v}_2 = \begin{bmatrix} 3/\sqrt{11} \\ 1/\sqrt{11} \\ -1/\sqrt{11} \end{bmatrix}$

1c. Find x and y which make $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ an orthogonal basis of $\mathbf{R}^3$. (Use good linear algebra techniques; your answer will involve a RREF).

we need $v_1 \perp v_3$; this means $3\cdot 1 - 4x + 5y = 0$, or, $-4x+5y=-3$

we need $v_2 \perp v_3$; this means $3\cdot 1 + 1x - 1y = 0$, or, $x - y = -3$

The corresponding augmented matrix is $\left[\begin{array}{cc|c} -4 & 5 & -3 \\ 1 & -1 & -3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & -18 \\ 0 & 1 & -15 \end{array}\right]$ so $\vec{v}_3 = \begin{bmatrix} 1 \\ -18 \\ -15 \end{bmatrix}$

1d. Use the formulas developed in class for orthogonal bases to find α_2 for which $\mathbf{s} = \alpha_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3$. (You do *not* have to find α_1 and α_3.)

$$\alpha_2 = \frac{\vec{s}\cdot\vec{v}_2}{\vec{v}_2\cdot\vec{v}_2} = \frac{-33+18-51}{3^2+1^2+1^2} = \frac{-66}{11} = -6$$

2. If $A = \begin{bmatrix} 2 & 3 & 3 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 1 & 5 & -6 \\ 2 & 1 & 1 & 2 \end{bmatrix}$ then RREF(A) is $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Find a basis for each of the following. Write vectors horizontally where appropriate.

2a. Col(A) $\left\{ \begin{bmatrix} 2 \\ 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \\ 1 \end{bmatrix} \right\}$

2b. Col(R) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

2c. Row(A) solution 1: since Row(A) = Row(R) and a basis for Row(R) is $\{[1\,0\,0\,1], [0\,1\,0\,2], [0\,0\,1\,-2]\}$ ← this is a basis of Row(A)

2d. Row(R) $\{[1\,0\,0\,1], [0\,1\,0\,2], [0\,0\,1\,-2]\}$

Soln 2: Row(A) = Col(A^T). RREF(A^T) shows cols 1, 2, 4 are pivot cols $\therefore$ a basis is rows 1, 2, AND 4 of A, and you could use $\{\vec{r}_1\,\vec{r}_2\,\vec{r}_4\}$

no a basis for Row(R) b/c Row(A) = Row(R)!

2e. Express $\mathbf{r}_3$ (ie, row 3) of A as a linear combination $\mathbf{r}_3 = x\mathbf{r}_1 + y\mathbf{r}_2 + z\mathbf{r}_4$. of the other three rows of A. (Hint: you will be on familar ground if you write the vectors vertically to solve the problem; find x y and z. Or explain why there are no such scalars. Use good linear algebra methods)

solve: $\begin{bmatrix} 2 \\ 1 \\ 5 \\ -6 \end{bmatrix} = x\begin{bmatrix} 2 \\ 3 \\ 3 \\ 2 \end{bmatrix} + y\begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix} + z\begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$; aug matrix representing this is

$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 2 \\ 3 & 2 & 1 & 1 \\ 3 & 1 & 1 & 5 \\ 2 & 3 & 2 & -6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \vec{r}_3 = 3\vec{r}_1 - 4\vec{r}_2 + 0\vec{r}_4$

(note well: this shows why $\{\vec{r}_1, \vec{r}_2, \vec{r}_3\}$ can NOT be a basis of Row(A), as the set is NOT L. I. !!)