1. Suppose that $\mathbf{v}_1 = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} 1 \\ x \\ y \end{bmatrix}$; let $\mathbf{s} = \begin{bmatrix} -11 \\ 18 \\ 51 \end{bmatrix}$.

1a. Explain why $\mathbf{v}_1 \perp \mathbf{v}_2$.

1b. Find a unit vector in the direction of $\mathbf{v}_2$.

1c. Find x and y which make $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ an orthogonal basis of $\mathbf{R}^3$. (Use good linear algebra techniques; your answer will involve a RREF).

1d. Use the formulas developed in class for orthogonal bases to find α_2 for which $\mathbf{s} = \alpha_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3$. (You do *not* have to find α_1 and α_3.)

2. If $A = \begin{bmatrix} 2 & 3 & 3 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 1 & 5 & -6 \\ 2 & 1 & 1 & 2 \end{bmatrix}$ then RREF(A) is $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Find a basis for each of the following. Write vectors horizontally where appropriate.

2a. Col(A)

2b. Col(R)

2c. Row(A)

2d. Row(R)

2e. Express $\mathbf{r}_3$ (ie, row 3) of A as a linear combination $\mathbf{r}_3 = x\mathbf{r}_1 + y\mathbf{r}_2 + z\mathbf{r}_4$. of the other three rows of A. (Hint: you will be on familar ground if you write the vectors vertically to solve the problem; find x y and z. Or explain why there are no such scalars. Use good linear algebra methods)