Consider the following matrices A and corresponding reduced row echelon form A_{rref}:

$$A = \begin{bmatrix} 1 & -2 & 4 & -7 & -10 & -11 & -12 \\ 2 & -4 & 9 & -17 & -23 & -26 & -30 \\ -5 & 10 & -18 & 29 & 45 & 47 & 51 \\ -4 & 8 & -19 & 37 & 51 & 56 & 71 \end{bmatrix} \qquad A_{rref} = \begin{bmatrix} 1 & -2 & 0 & 5 & 0 & 5 & 0 \\ 0 & 0 & 1 & -3 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

1a) Find a basis for Row A.

1b) Find a basis for Nul A.

1c) Find a basis for Col A.

1d) Find a basis for Row A_{rref}.

1e) Find a basis for Nul A_{rref}.

1f) Find a basis for Col A_{rref}.

1g) What is the rank of A?

2) Consider the set $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ where $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$.

2a) Explain why $\mathcal{B}$ cannot be a basis for $\mathbf{R}^3$.

2b) Is $\mathcal{B}$ a linearly independent set? Explain.

2c) Let $\mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. What relationship(s) among a, b, and c must be satisfied in order for $\mathbf{v}$ to be in the subspace H of $\mathbf{R}^3$ of which $\mathcal{B}$ *is* a basis?

2d) Expand $\mathcal{B}$ to a basis $\hat{\mathcal{B}}$ of $\mathbf{R}^3$. Explain how you found your third vector.

3) Let H be the subspace of $\mathbf{R}^4$ consisting of all vectors of the form $\mathbf{v} = \begin{bmatrix} 2a+3b+5c \\ 10c+7b+3a \\ 5(2b+c-a) \\ 3a+4b+10c+4a+4b+5c \end{bmatrix}$, where a, b, c, and d are arbitrary. Find a basis for H. Be *very* careful to make sure you have a basis!

4) Suppose $T : \mathbf{P}_4 \to \mathbf{P}_4$ is the linear transformation satisfying $T(p(x))$ = the second derivative of $p(x)$. Explicitly describe the kernel of T. Give a basis for it.

5) Let $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 2 \\ 2 & 0 & 0 \end{bmatrix}$ be the change of basis matrix from some basis $\mathcal{B}$ to $\mathcal{C} = \{t^2 + 2t + 3, \quad 3t + 2, \quad 1\}$, where these are bases of $\mathbf{P}_2$.

5a) Explicitly find the polynomials in $\mathcal{B}$.

5b) Suppose $[\mathbf{u}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 4 \\ 10 \end{bmatrix}$. Find $[\mathbf{u}]_{\mathcal{C}}$.

5c) Explicitly write out the polynomial $\mathbf{u}$.

5d) Suppose $[\mathbf{w}]_{\mathcal{C}} = \begin{bmatrix} 6 \\ -4 \\ 16 \end{bmatrix}$. Find $[\mathbf{w}]_{\mathcal{B}}$.

5e) Explicitly write out the polynomial $\mathbf{w}$.

6) Let $A = \begin{bmatrix} 6 & 0 & 0 \\ -4 & 8 & 10 \\ 8 & 3 & 7 \end{bmatrix}$

6a) Find the characteristic polynomial of A.

6b) Find all the eigenvalues of A.

6c) Find a basis of the eigenspace corresponding to the smallest of your eigenvalues.

7a) Give an example of an invertible 2x2 matrix which has 4 as an eigenvalue and the eigenspace is all of $\mathbf{R}^2$.

7b) Give an example of a 2x2 matrix whose null space is all of $\mathbf{R}^2$.

7c) Explain why you can't find an *invertible* matrix in 7b.

8) Suppose A is a 5x8 matrix with at least 2 pivot columns. What are the maximum and minimum dimensions of each of the following?

8a) Row(A)

8b) Col (A)

8c) Null (A)