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I_____II_____III_____IV_____V_____VI_____VII______VIII______ TOTAL ___________

March 25, 2005

Mathematics 206a
Multivariable Calculus
Examination #3

Mr. Haines

(15) I. Consider the path $\boldsymbol{f} : [0,\infty) \subset \mathbb{R} \to \mathbb{R}^2$ with $\boldsymbol{f}(t) = \left(e^{-t}\cos t, e^{-t}\sin t\right)$.

A. Sketch a graph of $\boldsymbol{f}$.

B. Give an integral that computes the total length of this path.

C. Calculate the value of this integral.

(10) II. Let C be the curve along the graph of $y = x^3 - x^2$ between the points (0, 0) and (1, 0).

A. Give a parametrization for C .

B. Set up but <u>do not evaluate</u> the line integral $\int_C (x+y)\,dL$.

(10) III. Evaluate the line integral $\int_C \boldsymbol{F}(x,y,z) \bullet d\boldsymbol{x}$ where $\boldsymbol{F}(x,y,z) = (y,z,x)$ and

A) C is the straight line segment from (0,0,0) to (1,2,3) .

B) C is the straight line segment from (1,2,3) to (0,0,0) .

(15) IV. Evaluate: $\int_0^1\int_0^z\int_0^y z\,dx\,dy\,dz$.

(15) V. Let M be the triangular surface in the plane $\frac{x}{2} + y + z = 1$ that is cut off by the three coordinate planes. (M lies in the first octant, where $x \geq 0, y \geq 0, \text{and } z \geq 0$.)

A. Give a parametrization for the surface M.

B. Set up, but <u>do not evaluate</u>, an iterated integral that gives the area of M.

C. Set up, but <u>do not evaluate</u>, an iterated integral that gives the surface integral of the vector field **F** (x, y, z) = (0, x, 0) over the surface M.

(10) VI. Let W be the solid bounded by the planes x = 0, y = 0, and z = 0, above the surface $z^2 = x^2 + y^2$ and below the sphere $x^2 + y^2 + z^2 = 16$. Use a change of variables to spherical coordinates to set up, but do not evaluate, an iterated integral that computes $\iiint_W x\,dV$.

(10) VII. Evaluate this double integral by converting to polar coordinates:

$\iint_R x\,dA$, where R is the region in the first quadrant bounded by the lines x = 0, y = x, and the circle $x^2 + y^2 = 9$.

(15) VIII. Let $\mathbf{f} : R \subset \Re^2 \to \Re^3$ be defined by $\boldsymbol{f}(s,t) = (s, t^2, s^2 t)$ with R = [0,1] x [0, 2]. Let M be the surface parametrized by $\boldsymbol{f}$.

A. Set up, but <u>do not evaluate</u>, an iterated integral that gives $\sigma(M)$, the surface area of M.

B. Let $\boldsymbol{g} : \mathbb{R}^3 \to \mathbb{R}$ be defined by $\boldsymbol{g}(x,y,z) = \dfrac{1}{\sqrt{1+x^2+y^2}}$

Set up, but <u>do not evaluate</u>, an iterated integral that gives $\iint_M \boldsymbol{g}\, d\sigma$

C. Let $\boldsymbol{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $\boldsymbol{F}(x,y,z) = (x, -y, 6)$

Set up, but <u>do not evaluate</u>, an iterated integral that gives $\iint_M \boldsymbol{F} \cdot \boldsymbol{n}\, d\sigma$