Given the vector field \( \mathbf{F}(x, y, z) = (2xyz, x^2z + 2yz, x^2y + y^2) \),

A) Prove that \( \mathbf{F} \) is path independent on \( \mathbb{R}^3 \) by finding a potential function for \( \mathbf{F} \).

B) If \( C \) is a path in \( \mathbb{R}^2 \) parametrized by \( \mathbf{e}(t) = \left( \frac{t^3}{3}, \sin^2 \frac{\pi t}{2}, 2t \right) \) with \( 0 \leq t \leq 1 \), calculate the path integral \( \int_C \mathbf{F} \cdot d\mathbf{x} \).