LINEAR ALGEBRA EXAM 3 March 24, 2004

Name: ______________________________

While the final answer is important, you earn points for all the work leading to that answer, as well as the answer itself. Show all your steps clearly so you will be eligible for the most partial credit. Good luck!

1.) (15 pts.) Determine whether the set of polynomials $\{3+7t, 5+t-2t^3, t-2t^2, 1+16t-6t^2+2t^3\}$ forms a basis for $\mathbb{P}_3$. Justify your conclusion.

2.) (10 pts.) Find the coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ of $\mathbf{x} = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$ relative to the basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \right\}$.

3.) (10 pts.) For the subspace $\left\{ \begin{bmatrix} s-2t \\ s+t \\ 3t \end{bmatrix} : s, t \text{ in } \mathbb{R} \right\}$,

a.) find a basis, and

b.) state the dimension.

4.) (15 pts.) Given the vectors $\mathbf{r} = \begin{bmatrix} 3 \\ -5 \\ -1 \end{bmatrix}$ and $\mathbf{s} = \begin{bmatrix} 6 \\ -7 \\ 3 \end{bmatrix}$, compute $\left(\frac{\mathbf{r} \cdot \mathbf{s}}{\mathbf{r} \cdot \mathbf{r}} \right) \mathbf{r}$.

5.) (10 pts.) Find the characteristic polynomial and the eigenvalues of the matrix $\begin{bmatrix} 5 & -3 \\ -4 & 3 \end{bmatrix}$.

6.) (10 pts.) Let A be an $n \times n$ matrix. **True or False:** An eigenspace of A is a null space of a certain matrix. *If True:* explain why in detail. *If False:* explain why in detail, and/or provide a counterexample, that is, an example to show when the statement is false.

7.) (15 pts.) A scientist solves a nonhomogeneous system of ten linear equations in twelve unknowns and finds that three of the unknowns are free variables. Can the scientist be certain that, if the right sides of the equations are changed, the new nonhomogeneous system will have a solution? Discuss.

8.) (15 pts.) Diagonalize the matrix $A = \begin{bmatrix} -7 & -16 & 4 \\ 6 & 13 & -2 \\ 12 & 16 & 1 \end{bmatrix}$, using the information that the eigenvalues of A are $\lambda = -3, 5$.