Name:______________________________

Mathematics 205
Exam II
March 23, 2012

Problem	Possible	Actual
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

You must show all work to receive credit.
No electronic devices other than calculators are permitted.
Give exact answers (such as $\ln 5$ or e^2) unless requested otherwise.

1. Let $\mathcal{K}$ be the set of 3×3 skew-symmetric matrices. Skew-symmetric matrices satisfy $A = -A^T$.

   (a) Give three examples of 3×3 matrices in $\mathcal{K}$.

   (b) Show that $\mathcal{K}$ is a vector space. You may use the fact that all 3×3 matrices form a vector space.

   (c) Find a basis for $\mathcal{K}$.

2. Recall from calculus that

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}.$$

We may apply this formally to $n \times n$ matrices to define

$$\sin A = \sum_{n=0}^{\infty} \frac{(-1)^n A^{2n+1}}{(2n+1)!}.$$

Compute $\sin\left(\begin{bmatrix} \pi/2 & \pi \\ 0 & \pi/6 \end{bmatrix}\right)$. Hint: Diagonalize A.

3. According to Kepler's first law, a comet should have an elliptic, parabolic, or hyperbolic orbit. In suitable polar coordinates, the position (r, θ) of a comet satisfies an equation of the form $r = \beta + e(r\cos\theta)$ where β is a constant and e is the eccentricity of the orbit. Suppose the following 5 places are observed:

$$(3, .88), (2.3, 1.1), (1.65, 1.42), (1.25, 1.77), (1.01, 2.14).$$

Using a least-squares approximation, determine the type of orbit. If $e < 1$ the orbit will be an ellipse. If $e = 1$ the orbit will be parabolic. If $e > 1$ the orbit will be hyperbolic.

4. Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$.

(a) Are these vectors linearly independent? Explain.

(b) Are these any of vectors orthogonal? Explain.

(c) Let $W = \text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$. Find an orthogonal basis for W.

5. The set of 3×3 matrices form a vector space.

   (a) Let $\mathcal{D}_3$ be the set of 3×3 matrices with determinant being 1.

      i. Give three examples elements in $\mathcal{D}_3$.

      ii. Is $\mathcal{D}_3$ a vector space?

   (b) Let $\mathcal{T}_3$ be the set of 3×3 matrices with trace 0. Recall that the trace of a matrix is the sum of the diagonal entries.

      i. Give three examples of elements in $\mathcal{T}_3$

      ii. Is $\mathcal{T}_3$ a vector space?