MATH 205 EXAM 2 March 23, 2007

Name: _____

Your grade is based on correctness, completeness, and clarity on each exercise. You may use a calculator, but no notes, books, or other students. Good luck!

1.) (15 pts.) Use the Invertible Matrix Theorem to answer the following questions. In each of them, assume the matrix A is $n \times n$. Also, for each of them, fully explain your reasoning.

a.) (5 pts.) If there is an $n \times n$ matrix D for which $AD = I$, is there also an $n \times n$ matrix C for which $CA = I$?

b.) (5 pts.) If A^T is not invertible, is A invertible?

c.) (5 pts.) If there is a vector $\mathbf{b}$ in $\mathbb{R}^n$ for which the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent, can the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ be one-to-one?

2.) (15 pts.) Given

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ -1 & -3 & 2 & 4 & -8 \\ -2 & -1 & -6 & -7 & 9 \\ 5 & 6 & 8 & 7 & -5 \end{bmatrix},$$

compute a basis for Col A and a basis for Nul A. What is the dimension of Col A? What is the dimension of Nul A? What is rank A?

3.) (15 pts.) In each of the following, assume the matrix A is 3×3. Also, for each of them, fully explain your reasoning.

a.) (5 pts.) Why is $\det A^T A \geq 0$?

b.) (5 pts.) Is $\det(4A) = 4 \det A$?

c.) (5 pts.) If two rows of A are the same, then what is $\det A$?

4.) (15 pts.) Compute all eigenvalues of the matrix A below. Then, for each eigenvalue, find a basis for its corresponding eigenspace.

$$A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$$

5.) (15 pts.) In each of the following, assume the matrix A is $n \times n$. Also, for each of them, fully explain your reasoning.

a.) (5 pts.) If A is diagonalizable, must A have n distinct eigenvalues?

b.) (5 pts.) An eigenspace of A is the null space of which matrix?

c.) (5 pts.) Construct a nonzero 2×2 matrix that is diagonalizable but not invertible. (*Hint:* what must be true of the eigenvalues in such a matrix?)

6.) (15 pts.) Consider the set of vectors

$$\{\mathbf{x}, \mathbf{y}, \mathbf{z}\} = \left\{ \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \begin{bmatrix} -1/2 \\ 1/2 \\ 1/2 \\ -1/2 \end{bmatrix}, \begin{bmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{bmatrix} \right\}.$$

a.) (5 pts.) Show that $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is an orthogonal set.

b.) (5 pts.) Show that $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is an orthonormal set.

c.) (5 pts.) Compute the projection of $\mathbf{y}$ on the vector $\mathbf{u} = \begin{bmatrix} -2 \\ 6 \\ 4 \\ 2 \end{bmatrix}$.

7.) (10 pts.) A certain copy machine is always either working or broken (not working). If it is working today, there is a 70% chance that it will be working tomorrow. If it is broken today, there is a 50% chance that it will be broken tomorrow.

a.) (5 pts.) Assuming the copy machine is working today, what is the probability that it is working in two days?

b.) (5 pts.) What is the long-term probability that the copy machine will be working on any given day?