Given the vector field \( \mathbf{F}(x, y) = (1 + y^3, \, 3xy^2) \),

A) Prove that \( \mathbf{F} \) is path independent on \( \mathbb{R}^2 \) by finding a potential function for \( \mathbf{F} \).

B) If \( \mathbf{C} \) is a path in \( \mathbb{R}^2 \) parametrized by \( \mathbf{c}(t) = \left( \frac{t^3}{3}, \sin^2 \frac{\pi t}{2} \right) \) with

\[ 0 \leq t \leq 1, \text{ calculate the path integral } \int_{\mathbf{C}} \mathbf{F} \cdot d\mathbf{x}. \]